

CSC/MAT-220: HOMEWORK 1

DUE: 8/31/2018

Book Problems

Please do each of the following problems from the class book [2]:

3.9, 4.9, 5.9, 5.21, 6.2, 6.3, 7.14

Other Problems

I. The following problem is taken from Lay [1]: Use truth tables to verify that each of the following is a tautology. Parts i. and ii. are call *commutative laws*, parts iii. and iv. are *associative laws*, and parts v. and vi. are *distributive laws*.

i. $(p \wedge q) \Leftrightarrow (q \wedge p)$

ii. $(p \vee q) \Leftrightarrow (q \vee p)$

iii. $[p \wedge (q \wedge r)] \Leftrightarrow [(p \wedge q) \wedge r]$

iv. $[p \vee (q \vee r)] \Leftrightarrow [(p \vee q) \vee r]$

v. $[p \wedge (q \vee r)] \Leftrightarrow [(p \wedge q) \vee (p \wedge r)]$

vi. $[p \vee (q \wedge r)] \Leftrightarrow [(p \vee q) \wedge (p \vee r)]$

II. The following puzzle is taken from Smullyan [3]: There is an island of knights and knaves, where knights always tell the truth and knaves always lie. On one of my visits to the island I was introduced to three inhabitants A , B , and C . At least one of them was a knight, at least one of them was a knave, and one of them had a prize that I could have if I could determine which one had it. The three made the following statements:

Statement A : B doesn't have the prize.

Statement B : I don't have the prize.

Statement C : I have the prize.

Use these statements to determine who is a knight, a knave, and who has the prize.

III. The following story is taken from Smullyan [3]: Paradoxes go back to ancient times. One of the oldest paradoxes is Zeno's famous paradox, "proving" that motion is impossible. Suppose a body is to move from a point A to a point B . Starting from A , before it can get to B it must get to a point A_1 halfway between A and B . This is the first step. Before it can get from A_1 to B it must pass a point A_2 halfway between A_1 and B . This is the second step. The third step is to go from A_2 to a point A_3 midway between A_2 and B , and so forth. After any finite number of such steps it has not reached B . Thus no finite number of steps can suffice for the body to reach B , and so motion is impossible.

Note that if an argument leads to a false conclusion there must be a first false step in the argument. What is wrong with Zeno's argument?

References

- [1] S. R. Lay, *Analysis: with an introduction to proof*, 4th ed., Pearson Education, Upper Saddle River, NJ, 2005.
- [2] E. R. Scheinerman, *Mathematics: A discrete introduction*, 3rd ed., Brooks/Cole, Boston, MA, 2013.
- [3] R. M. Smullyan, *The Gödelian puzzle book*, Dover, Mineola, NY, 2013.