1.1 and 1.2 Differential Equation Terminology

$y = f(x)$ is an unknown function.

$y' = \frac{dy}{dx} = f'(x)$ is the rate of change of y with respect to (wrt) x.

Modeling
physical situation $\Rightarrow$ relationship between rates of change $\Rightarrow$ equation with derivatives.
This results in a *differential equation (DE)* which is a mathematical model of the physical situation.
If there are *initial conditions* (known starting values of the unknown function and derivatives), then the differential equation to together with the initial conditions is called an *initial value problem (IVP)*.

Classify the DE
The *order of the DE* is the order of the highest derivative in the equation.
We will also develop many special categories of equations, such as, separable, linear, homogeneous, autonomous,...

Solution Terminology
Usually there are infinitely many solutions.

A *(particular) solution* is a function $y = f(x)$ that satisfies the DE, i.e., if you substitute it in and compute, the equation is shown to be true.

A general solution of an nth order DE is a solution including n arbitrary constants. We call it *the general solution* if it can be proved that the solution formula describes all possible solutions.

A *solution of an IVP* is a function $y = f(x)$ that satisfies the DE and satisfies the initial conditions. Usually this solution is unique.

Solution Techniques (find a function that satisfies the DE)
1. theoretical -- does it exist? is it unique? over what domain?

2. numerical -- compute accurate estimates of the function values and graph.

3. qualitative -- use mathematical analysis of the DE to determine the important "features" of the graph of the solution and the long-term behavior of the solution.

4. analytic (or symbolic) -- find a "formula" that satisfies the equation
      (a) elementary formula -- *any* combination of the standard functions: polynomials, e^x, ln(x), trig functions, inverse trig, etc.
      (b) implicit formula -- an equation, *not* involving derivatives, that describes y but is not solved for y.
      (c) series or integral formulas -- describe (possibly non-elementary) functions.