MAT-235: Homework 2

Due: 9/14/2018

Book Problems

Please do each of the following problems from the class book [1]:

Section 2.1: 13 and 32

Section 2.2: 4, 7 (Make rough sketch based on information from phase diagram, no computer necessary), and 29

Section 2.3: 7 and 11

Section 3.1: 14, 17, and 34

Section 3.2: 4, 8, 21, 27, 30

Section 3.3: 9, 17, 21, 23, 30

Other Problems

Consider the homogeneous linear nth-order differential equation:

$$a_n(x)y^n + a_{n-1}(x)y^{n-1} + \cdots + a_1(x)y' + a_0(x)y = 0, \tag{1}$$

where $a_i(x)$, $i = 0, 1, \ldots, n$, are continuous and $a_n(x) \neq 0$ on some interval I.

I. Write a proof for the following theorem and corollary:

Theorem. *Let $y_1, y_2, \ldots, y_k$ be solutions of (1) on an interval I. Then the linear combination*

$$y = c_1y_1 + c_2y_2 + \cdots + c_ky_k$$

where c_i, $i = 1, 2, \ldots, k$ are arbitrary constants, is also a solution on the interval.

Corollary. *The constant function $y \equiv 0$ is always a solution of (1).*

II. Write a proof for the following theorem:

Theorem. *Let $y_1, y_2, \ldots, y_n$ be linearly independent solutions of (1) on an interval I. Then for any solution Y of (1), there exists constants $c_1, c_2, \ldots, c_n$ such that*

$$Y = c_1y_1 + c_2y_2 + \cdots + c_ny_n.$$

Hint: Use [1, Theorem 3, Section 3.2] and the proof of [1, Theorem 4, Section 3.1] for guidance.

III. The following problem was taken from [2]: Show graphically that

$$f_1(x) = x^2 \text{ and } f_2(x) = x|x|$$

are linearly independent on $-\infty < x < \infty$. In addition, show that $W(f_1(x), f_2(x)) \equiv 0$. Why does this not contradict [1, Theorem 3, Section 3.1]?

References

[1] C. H. Edwards, D. E. Penny, and D. T. Calvis, *Differential equations and boundary value problems, computing and modeling*, 5th ed., Pearson Education, Upper Saddle River, NJ, 2019.

[2] D. G. Zill, *A first course in differential equations with applications*, 2nd ed., PWS Publishers, Boston, MA, 1982.