MAT-235: Homework 3

Due: 9/28/2018

Book Problems

Please do each of the following problems from the class book [1]:

Section 3.4: 4, 10, and 15
Section 3.5: 20, 30, 32, 39, and 43
Section 3.6: 4, 5, 11, 19, and 23
Section 3.8: 1, 2, 13, and 16

Other Problems

Consider the non-homogeneous linear nth-order differential equation:

$$a_n(x)y^n + a_{n-1}(x)y^{n-1} + \cdots + a_1(x)y' + a_0(x)y = f(x), \tag{1}$$

where $a_i(x)$, $i = 0, 1, \ldots, n$, are continuous and $a_n(x) \neq 0$ on some interval I.

I. Write a proof of the following theorem:

Theorem. *Let y_p be a given solution of (1) on an interval I and let $y_1, \ldots, y_n$ be a basis of solutions of the associated homogeneous equation on I. Then for any solution Y of (1) on I, there exists constants $c_1, c_2, \ldots, c_n$ such that*

$$Y(x) = c_1y_1(x) + c_2y_2(x) + \cdots + c_ny_n(x) + y_p(x)$$

for all x in I.

II. The following problem was taken from [2]: Use polynomial differential operators to form the auxiliary equation associated with the differential equation

$$y'' + 3y' + 2y = 4x^2.$$

Then, use the auxiliary equation to determine the complementary function and particular solution.

III. The following problem was taken from [2]: The complementary function for

$$y''' - 2y'' - y' + 2y = e^{3x}$$

is

$$y_c = c_1e^{-x} + c_2e^{x} + c_3e^{2x}.$$

Find variables u_1, u_2, and u_3 such that

$$y_p = u_1(x)e^{-x} + u_2(x)e^{x} + u_3(x)e^{2x}$$

is a particular solution.

References

[1] C. H. Edwards, D. E. Penny, and D. T. Calvis, *Differential equations and boundary value problems, computing and modeling*, 5th ed., Pearson Education, Upper Saddle River, NJ, 2019.

[2] D. G. Zill, *A first course in differential equations with applications*, 2nd ed., PWS Publishers, Boston, MA, 1982.