MAT-331: Algebra and Geometry of Complex Numbers

Due: Next Class

Instructions

Begin by reflecting on the following quote from [2].

> **Nicht einer mystischen Verwendung von $\sqrt{-1}$ hat die Analysis ihre wirklich bedeutenden Erfolge des letzten Jahrhunderts zu verdanken, sondern dem ganz natürlichen Umstande, dass man unendlich viel freier in der mathematischen Bewegung ist, wenn man die Grössen in einer Ebene statt nur in einer Linie variiren läßt (Analysis does not owe its really significant successes of the last century to any mysterious use of $\sqrt{-1}$, but to the quite natural circumstance that one has infinitely more freedom of mathematical movement if he lets quantities vary in a plane instead of only on a line) – (Leopold Kronecker, in [Kr].)**

Write your thoughts down on a piece of paper. Share your thoughts in a group. Then, in those same groups, do each of the following problems and turn in to me at the beginning of next class. Note that all definitions and results can be found in Sections 1.2–1.3 of [1].

I. Let $z = x+iy$ be a complex number. State the definition and provide formula for the following concepts:
   - Absolute value of z.
   - Argument of z.
   - Polar form of z.
   - Complex conjugate of z.

II. Show that complex multiplication can be viewed as a dilation and rotation.

III. Prove Proposition 1.3.

IV. Prove Proposition 1.5.

V. Prove Corollary 1.7.

References

[1] M. Beck, G. Marchesi, D. Pixton, and L. Sabalka, *A First Course in Complex Analysis*, vol. 1.54, Orthogonal Publishing, Ann Arbor, MI, 2002.

[2] Reinhold Remmert, *Theory of Complex Functions*, Springer-Verlag, New York, NY, 1991.