MATH-110 **Name:** ____________________

Spring 2023

Exam 1 Solution

January 25 **Pledge:** ____________________

Each question topic and point value is recorded in the tables below. Note that this exam must be completed within the 50 minutes allotted during class. Also, you must work without any external resources, which includes no notes, calculator, nor any equivalent software. You must show an appropriate amount of work to justify your answer for each problem. If you run out of room for a given problem, you may continue your work on the back of the page. By writing your name and signing the pledge you are stating that you understand the rules outlining this exam.

Scoring Table

Question	Points	Score
1	5	
2	10	
3	6	
4	9	
5	6	
Total:	36	

Topics Table

Question	Topic
1	Log and Exponential Equations
2	Limits via Graphs
3	Limit Properties
4	Computing Limits
5	Continuity

1. (a) (2 points) Solve the following exponential equation. Fully simplify your answer.

$$2^{x^2} = 4^{4-x}$$

Solution: Note that $4^{4-x} = \left(2^2\right)^{4-x} = 2^{8-2x}$. Therefore, the given equation can be written as $2^{x^2} = 2^{8-2x}$. Therefore, the solution satisfies

$$x^2 = 8 - 2x \Longrightarrow x^2 + 2x - 8 = 0.$$

Factoring gives us $x^2 + 2x - 8 = (x+4)(x-2)$. Therefore, the solution is $x = 2, -4$.

(b) (3 points) Solve the following log equation. Fully simplify your answer.

$$2\ln(x) - \ln(x+2) = 0$$

Solution: We rewrite the given equation as follows

$$\ln\left(\frac{x^2}{x+2}\right) = \ln(1).$$

Therefore, the solution satisfies

$$\frac{x^2}{x+2} = 1 \Longrightarrow x^2 = x + 2 \Longrightarrow x^2 - x - 2 = 0.$$

Factoring gives us $x^2 - x - 2 = (x-2)(x+1)$. Therefore, the potential solution is $x = -1, 2$; however, $x = -1$ is not in the domain of $\ln(x)$. So, the only solution is $x = 2$.

2. (10 points) For the function f graphed in the accompanying figure, compute each of the indicated values. If the limit does not exist, write DNE and explain why the limit does not exist.

a. $\lim_{x \to 0^-} f(x)$

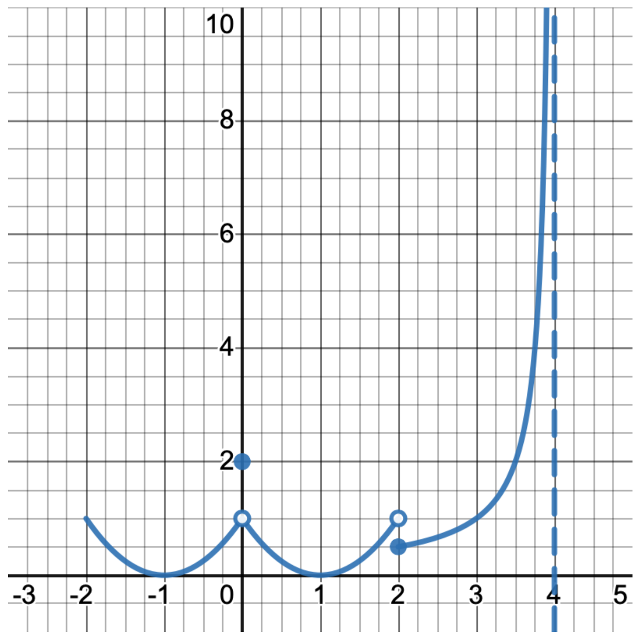
Solution: $\lim_{x \to 0^-} f(x) = 1$

b. $\lim_{x \to 0^+} f(x)$

Solution: $\lim_{x \to 0^+} f(x) = 1$

c. $\lim_{x \to 0} f(x)$

Solution: $\lim_{x \to 0} f(x) = 1$



d. $\lim_{x \to 2^-} f(x)$

Solution: $\lim_{x \to 2^-} f(x) = 1$

e. $\lim_{x \to 2^+} f(x)$

Solution: $\lim_{x \to 2^+} f(x) = 0.5$

f. $\lim_{x \to 2} f(x)$

Solution: $\lim_{x \to 2} f(x) = DNE$, since the left and right sided limits are not equal.

g. $\lim_{x \to 4^-} f(x)$

Solution: $\lim_{x \to 4^-} f(x) = +\infty$

h. $f(0)$

Solution: $f(0) = 2$

i. $f(2)$

Solution: $f(2) = 0.5$

j. $f(4)$

Solution: $f(4) = undefined$, since $x = 4$ is a vertical asymptote of f.

3. (6 points) Suppose that

$$\lim_{x\to 0} f(x) = 3 \text{ and } \lim_{x\to 0} g(x) = 5.$$

Compute the indicated limits below. In your work, make it clear which limit properties you are using.

a. $\lim_{x\to 0} 3f(x) - g(x) + 1$

Solution:

$$\begin{aligned}\lim_{x\to 0} 3f(x) - g(x) + 1 &= \lim_{x\to 0} 3f(x) - \lim_{x\to 0} g(x) + \lim_{x\to 0} 1 \\ &= 3\lim_{x\to 0} f(x) - 5 + 1 \\ &= 3(3) - 5 + 1 = 5.\end{aligned}$$

b. $\lim_{x\to 0} \dfrac{f(x) + \sqrt{x}}{f(x)g(x) - 6}$

Solution:

$$\begin{aligned}\lim_{x\to 0} \frac{f(x) + \sqrt{x}}{f(x)g(x) - 6} &= \frac{\lim_{x\to 0} f(x) + \sqrt{x}}{\lim_{x\to 0} f(x)g(x) - 6} \\ &= \frac{\lim_{x\to 0} f(x) + \lim_{x\to 0} \sqrt{x}}{\lim_{x\to 0} f(x) \lim_{x\to 0} g(x) - \lim_{x\to 0} 6} \\ &= \frac{3 + 0}{(3)(5) - 6} = \frac{3}{9} = \frac{1}{3}.\end{aligned}$$

4. (9 points) Compute the indicated limits below. Show your work.

a. $\lim_{x \to 4} \frac{x-4}{\sqrt{x}-2}$

Solution:

$$\begin{aligned}\lim_{x \to 4} \frac{x-4}{\sqrt{x}-2} &= \lim_{x \to 4} \frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} \\ &= \lim_{x \to 4} \frac{(x-4)(\sqrt{x}+2)}{x-4} \\ &= \lim_{x \to 4} \sqrt{x}+2 = 4.\end{aligned}$$

b. $\lim_{x \to -1} \frac{x^2-x-2}{x+1}$

Solution:

$$\begin{aligned}\lim_{x \to -1} \frac{x^2-x-2}{x+1} &= \lim_{x \to -1} \frac{(x+1)(x-2)}{x+1} \\ &= \lim_{x \to -1} (x-2) = -3.\end{aligned}$$

c. $\lim_{x \to 1} \frac{x-1}{x^2+x+1}$

Solution:

$$\lim_{x \to 1} \frac{x-1}{x^2+x+1} = \frac{0}{3} = 0.$$

5. (6 points) Identify the x values where each indicated function below is not continuous. State which part of the limit definition for continuity is being violated.

a. $f(x) = \dfrac{x}{x-1}$

Solution: Note that $f(1)$ is undefined and $\lim_{x\to 1^+} f(x) = +\infty$. Hence, f is discontinuous at $x = 1$ since $x = 1$ is a vertical asymptote of f.

b. $f(x) = \begin{cases} 2x+3, & x \leq 4 \\ 8 + \frac{16}{x}, & x > 4 \end{cases}$

Solution: Note that $(2x+3)$ is continuous for all $x \leq 4$ and $8+\frac{16}{x}$ is continuous for all $x > 4$. Therefore, the only potential point of discontinuity is at $x =$. Note that $f(4) = 11$ but $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+}(8 + 16/x) = 12$. Therefore, f is discontinuous at $x = 4$.