- Math-140 -

Last time, we left off trying to find the derivative of

$$f(x) = \frac{3x^2 + 4x - 8}{x^{3/2}} = 3x^{1/2} + 4x^{-1/2} - 8x^{-3/2}$$

$$f'(x) = \frac{3}{2}x^{-1/2} - 2x^{-3/2} + 12x^{-5/2}.$$

Today, we continue our discussion of derivative rules for transcendental functions.

$$\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1$$

$$\lim_{\theta \to 0} \frac{\cos(\theta) - 1}{\theta} = 0$$

Example - $\left(\frac{d}{dx} \sin x = \cos x\right)$.

$$\frac{d}{dx}\sin(x) = \lim_{\Delta x \to 0} \frac{\sin(x+\Delta x) - \sin(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin(x)\cos(\Delta x) + \cos(x)\sin(\Delta x) - \sin(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \sin(x)\frac{(\cos(\Delta x) - 1)}{\Delta x} + \lim_{\Delta x \to 0} \cos(x)\frac{\sin(\Delta x)}{\Delta x}$$

$$= \cos(x).$$

$$e = \lim_{u \to 0} (1+u)^{1/u}$$

$$\ln(e) = 1$$

Example - $\left(\frac{d}{dx}\ln(x) = \frac{1}{x}\right)$

$$\frac{d}{dx}\ln(x) = \lim_{\Delta x \to 0} \frac{\ln(x+\Delta x) - \ln(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{1}{\Delta x} \cdot \ln\left(\frac{x+\Delta x}{x}\right)$$

% let $u = \frac{\Delta x}{x}$

$u \to 0$ as $\Delta x \to 0$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \ln\left(1 + \frac{\Delta x}{x}\right)$$

$$= \lim_{u \to 0} \frac{1}{u \cdot x} \ln(1+u)$$

$$= \frac{1}{x} \cdot \lim_{u \to 0} \frac{1}{u} \cdot \ln(1+u) = \frac{1}{x} \cdot \lim_{u \to 0} \ln\left((1+u)^{1/u}\right)$$

$$= \frac{1}{x} \cdot \ln\left(\lim_{u \to 0} (1+u)^{1/u}\right)$$

$$= \frac{1}{x} \cdot \ln(e) = \frac{1}{x}.$$

Table of Rules -

$$\frac{d}{dx} C = 0$$

$$\frac{d}{dx} cf(x) = cf'(x)$$

$$\frac{d}{dx} (f(x)+g(x)) = f'(x) + g'(x)$$

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} \sin(x) = \cos(x)$$

$$\frac{d}{dx} \cos(x) = -\sin(x)$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

$$\frac{d}{dx} e^x = e^x$$ % prove this later

Exercises -

Find the derivatives

a) $6\sqrt{x} + 5\cos(x)$

b) $x^2 + \sin(x) - e^x + \ln(x)$

c) $34\cos(x) + \ln(e^x)$

Find the tangent lines

a) tangent to $\sin(x)$ at $(0,0)$

b) tangent to e^x parallel to $2x + 3$

c) horizontal tangent line to $\sin(x)$