- Math - 140 -

Product Rule -

Let f, g be differentiable functions. Then, their product is a differentiable function and

$$\frac{d}{dx} f(x) \cdot g(x) = f'(x) g(x) + f(x) g'(x).$$

Proof -

$$\frac{d}{dx} f(x) \cdot g(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) g(x+\Delta x) - f(x) g(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) g(x+\Delta x) - f(x) g(x+\Delta x) + f(x) g(x+\Delta x) - f(x) g(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} g(x+\Delta x) \cdot \frac{f(x+\Delta x) - f(x)}{\Delta x} + \lim_{\Delta x \to 0} f(x) \cdot \frac{g(x+\Delta x) - g(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} g(x+\Delta x) \cdot \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} + \lim_{\Delta x \to 0} f(x) \cdot \lim_{\Delta x \to 0} \frac{g(x+\Delta x) - g(x)}{\Delta x}$$

$$= g(x) \cdot f'(x) + f(x) \cdot g'(x).$$

Q: Why is $\lim_{\Delta x \to 0} g(x+\Delta x) = g(x)$?

A: Differentiable functions are continuous

$$g'(c) = \lim_{\Delta x \to 0} \frac{g(c+\Delta x) - g(c)}{\Delta x}$$

$$= \lim_{x \to c} \frac{g(x) - g(c)}{x - c}$$

Since this limit must exist and be finite, it follows that $\lim_{x \to c} g(x) = g(c)$.

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Quotient Rule -

If f, g are differentiable and $g \neq 0$, then

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

Table of Rules -

$\frac{d}{dx} c = 0$

$\frac{d}{dx} cf(x) = cf'(x)$

$\frac{d}{dx}(f(x)+g(x)) = f'(x)+g'(x)$

$\frac{d}{dx} x^n = nx^{n-1}$

$\frac{d}{dx}\sin(x) = \cos(x)$

$\frac{d}{dx}\cos(x) = -\sin(x)$

$\frac{d}{dx}\ln(x) = \frac{1}{x}$

$\frac{d}{dx} e^x = e^x$

$\frac{d}{dx} f(x)\cdot g(x) = f'g + fg'$

$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'g - fg'}{g^2}$

Exercises -

Find the derivative of

a) $\tan(x)$

b) $\frac{x-1}{x^2+1}$

c) $e^x \cdot \ln(x)$

Find the tangent line

a) horizontal to x^2e^{-x}

b) tangent to $\cot(x)$ at $(\frac{\pi}{2}, 0)$