Math-140 - 2/7/2024

What did I do wrong?

a) $\frac{d}{dx}\sqrt{1+\sin^2(x)} = \frac{1}{2}\left(1+\sin^2(x)\right)^{-1/2} \cdot 2\sin(x)$

b) ~~The~~ Tangent line to $f(x) = \sin(x)$ at $(\pi/2, 1)$ is given by

$$y - 1 = \cos(x)\,(x - \pi/2)$$

Chain Rule Exercises:

a) $\frac{d}{dx}\cos^3\left(\frac{x}{x+1}\right) = 3\cos^2\left(\frac{x}{x+1}\right) \cdot \frac{d}{dx}\cos\left(\frac{x}{x+1}\right)$

$$= 3\cos^2\left(\frac{x}{x+1}\right) \cdot -\sin\left(\frac{x}{x+1}\right) \cdot \frac{d}{dx}\frac{x}{x+1}$$

$$= -3\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right) \cdot \frac{1(x+1) - x(1)}{(x+1)^2}$$

$$= -3\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right) \cdot \frac{1}{(x+1)^2}$$

b) $\frac{d}{dx}\sqrt{x}\tan^3(\sqrt{x}) = \frac{1}{2}x^{-1/2} \cdot \tan^3(\sqrt{x}) + x^{1/2} \cdot \frac{d}{dx}\tan^3(\sqrt{x})$

$$= \frac{1}{2}x^{-1/2} \cdot \tan^3(\sqrt{x}) + x^{1/2} \cdot 3\tan^2(\sqrt{x}) \cdot \frac{d}{dx}\tan(\sqrt{x})$$

$$= \frac{1}{2}x^{-1/2} \cdot \tan^3(\sqrt{x}) + x^{1/2} \cdot 3\tan^2(\sqrt{x}) \cdot \sec^2(\sqrt{x}) \cdot \frac{1}{2}x^{-1/2}$$

- Math-140 -

c) $\frac{d}{dx} \ln(\sin^2(x)+1) = \frac{1}{\sin^2(x)+1} \cdot \frac{d}{dx}(\sin^2(x)+1)$

$= \left(\frac{1}{\sin^2(x)+1}\right) \cdot 2\sin(x)\cdot\cos(x)$

$= \frac{2\sin(x)\cos(x)}{\sin^2(x)+1}$

d) Find the tangent line to $f(x)=\sqrt{25-e^{2x}}$ at $x=0$.

$f'(x) = \frac{1}{2}(25-e^{2x})^{-1/2} \cdot \frac{d}{dx}(25-e^{2x})$

$= \frac{1}{2}(25-e^{2x})^{-1/2} \cdot -e^{2x} \cdot 2$

$= \frac{-1}{2}(25-e^{2x})^{-1/2} \cdot 2e^{2x} = \frac{-e^{2x}}{\sqrt{25-e^{2x}}}$

$y-\sqrt{24} = \frac{-1}{\sqrt{24}}(x-0)$