- Math-140 - 2/9/2024 -

Explicit vs Implicit -

$$x^2 + y^2 = 1 \qquad \text{(implicit)}$$

$$y = \cos(\sqrt{x^2+1}) \qquad \text{(explicit)}$$

Implicit Differentiation -

If I gave you the function $y = \cos(\sqrt{x^2+1})$ and asked for the derivative dy/dx you would apply d/dx to both sides of the equation:

$$\frac{d}{dx} y = \frac{d}{dx} \cos(\sqrt{x^2+1})$$

$$= -\sin(\sqrt{x^2+1}) \cdot \frac{d}{dx} \sqrt{x^2+1}$$

$$= -\sin(x^2+1) \cdot \frac{1}{2}(x^2+1)^{-1/2} \cdot \frac{d}{dx}(x^2+1)$$

$$= -\sin(x^2+1) \cdot \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x$$

We proceed the same way with an implicit equation:

$$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx} 1$$

$y^2 = (y)^2$, y depends on x

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}.$$

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In general, it is not necessary to solve for y explicitly in terms of x in order to find dy/dx. For example,

$xy = 1$

$y = \frac{1}{x}$

$\frac{dy}{dx} = \frac{d}{dx} x^{-1} = -1x^{-2} = \frac{-1}{x^2}$

$xy = 1$

$\frac{d}{dx}(xy) = \frac{d}{dx}(1)$

$1 \cdot y + x \cdot \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{y}{x} = \frac{-1}{x^2}$

Exercises - Find dy/dx for each of the following

a) $5y^2 + \sin(y) = x^2$

b) $y^3 + y^2 - 5y - x^2 = -4$

c) Find slope of tangent line at given point

c) $3(x^2+y^2)^2 = 100xy$ at $(3,1)$

d) $x^3 + y^3 = 3xy$ at $(\frac{3}{2}, \frac{3}{2})$

Find where tangent line to $x^3+y^3 = 3xy$ is horizontal.