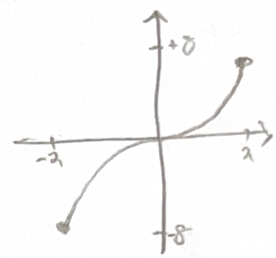
Math-140: 2/26/2024

Extrema -

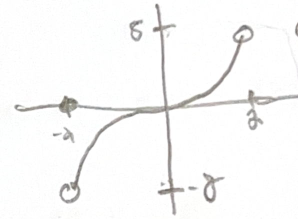
- f has an absolute max (min) on $[a,b]$ at c if $f(c) \geq f(x)$ ($f(c) \leq f(x)$) for all $x \in [a,b]$



On $[-2,2]$ $f(x)=x^3$ has absolute min at $c=-2$ ($f(-2)=-8$) and absolute max at $c=2$ ($f(2)=2$)

Extreme Value Theorem -

If f is continuous on $[a,b]$, then f has an absolute max and min on $[a,b]$



On $[-2,2]$, $f(x)=\begin{cases} x^3, & -2<x<2 \\ 0, & x=-2, x=2 \end{cases}$

does not have an absolute max or min

- f has a relative max (min) at c if there is a $\delta>0$ such that $f(c) \geq f(x)$ ($f(c) \leq f(x)$) for all $x \in [c-\delta, c+\delta]$.

$c=-1$ is a relative and absolute max

$c=-2$ is an absolute min

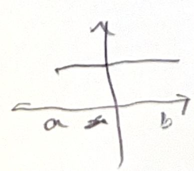
$c=1$ is a relative min.

Critical Numbers -

Relative extrema occur at critical numbers, i.e., places where $f'(c)=0$ or $f'(c)$ is undefined.

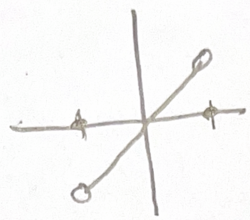
Q: When does a function have relative extrema?

Rolle's Theorem - Let f be continuous on $[a,b]$ and differentiable on (a,b). If $f(a)=f(b)$, then there is a $c \in (a,b)$ s.t. $f'(c)=0$

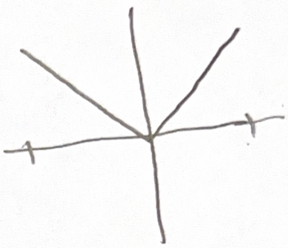


Q: Why does Rolle's theorem not apply?

a)



b)



Example - Let $f(x) = x^4 - 2x^2$

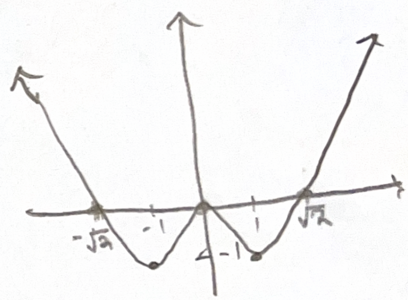
$= x^2(x^2 - 2)$

Note that $f(x) = 0$ when $x = 0, \ x = \pm\sqrt{2}$

Rolles theorem implies $\exists\, c_1 \in (-\sqrt{2}, 0)$

and $c_2 \in (0, \sqrt{2})$ s.t.

$f'(c_1) = f'(c_2) = 0$



Note that $f'(x) = 4x^3 - 4x$

$= 4x(x^2 - 1)$

$f'(x) = 0$ when $x = 0, \ x = \pm 1$

$f(-1) = f(1) = -1$

Example - Does ~~for~~ Rolle's theorem apply? If so, find all c s.t. $f'(c) = 0$.

a) $f(x) = -x^2 + 3x \quad ; \quad [0, 3]$

b) $f(x) = (x-1)(x-2)(x-3) \quad ; \quad [1, 3]$

c) $f(x) = \sin(x) \quad ; \quad [0, 2\pi]$

d) $f(x) = \dfrac{x^2 - 2x - 3}{x + 2} \quad ; \quad [-1, 3]$