- Math-140: 2/28/2024 -

Increasing/Decreasing -

(non-decreasing) (non-increasing)

Let (a,b) denote an open interval. Then, $f(x)$ is increasing (decreasing) on (a,b) if for all $x_1 < x_2$ on (a,b) we have

$$f(x_1) \le f(x_2) \qquad \left(f(x_1) \ge f(x_2) \right)$$

Prop - Suppose $f'(x) \ge 0$ for all x in (a,b), then $f(x)$ is increasing on (a,b).

Proof - Let $x_1 < x_2$ in (a,b). Then, there is a c between x_1 and x_2 such that

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = f'(c) \ge 0$$

Since $x_2 - x_1 > 0$, it follows that $f(x_2) - f(x_1) \ge 0$

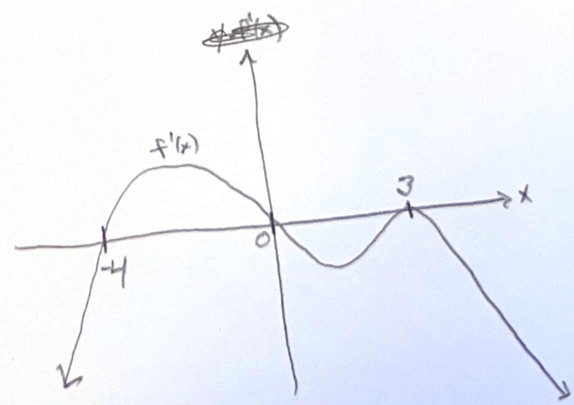
A similar result shows that if $f'(x) \le 0$ for all x in (a,b), then $f(x)$ is decreasing on (a,b).

Hence, we can use the derivative to identify the increasing/decreasing behavior of the function.

Example -

Increasing: $(-4, 0)$

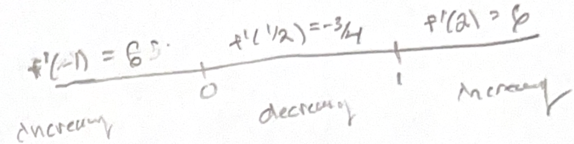
Decreasing: $(-\infty, -4) \cup (0, 3) \cup (3, \infty)$

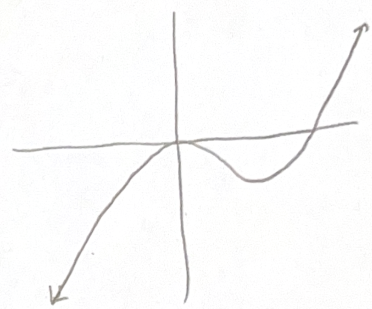


Note when switching from increasing to decreasing we have a critical number

__Example__ - Let $f(x) = x^3 - \frac{3}{2}x^2$. Note that $f'(x) = 3x^2 - 3x$
$= 3x(x-1)$

Hence, $f'(x) = 0$ when $x = 0, 1$





__Example__ - $f(x) = \dfrac{x^3+1}{x^2}$, $f'(x) = \dfrac{3x^2 \cdot x^2 - (x^3+1)\,2x}{x^4}$

$= x + \dfrac{1}{x^2}$ $\qquad = \dfrac{3x^4 - 2x^4 - 2x}{x^4}$

$= \dfrac{x^4 - 2x}{x^4} = 1 - \dfrac{2}{x^3}$

$f'(x) = 0$ when $x^4 - 2x = x(x^3 - 2) = 0$

$\Rightarrow x = 0, 2^{1/3}$