Homework 02 – Solutions

Math 140-002: Calculus I (Spring 2026)
Week 2 (Jan 19–Jan 23, 2026)

Relevant topics: Limits (one-sided/two-sided), limit laws, infinite limits, continuity, vertical asymptotes, ϵ–δ definition

1. **Problem.** Evaluate $\lim_{x\to 2}(3x-1)$.

   Solution. 5.

2. **Problem.** Evaluate $\lim_{x\to -1}\dfrac{x^2-1}{x+1}$.

   Solution. Factor to $x-1$ for $x \neq -1$, so the limit is -2.

3. **Problem.** Evaluate $\lim_{x\to 0}\dfrac{\sin(5x)}{x}$.

   Solution. 5.

4. **Problem.** Evaluate $\lim_{x\to 0}\dfrac{e^x-1}{x}$.

   Solution. 1.

5. **Problem.** Evaluate $\lim_{x\to 1}\dfrac{5x^2+1}{2x^2-3}$.

   Solution. -6.

6. **Problem.** Evaluate $\lim_{x\to\infty}\ln x$.

   Solution. $+\infty$.

7. **Problem.** State the ϵ–δ definition of $\lim_{x\to a} f(x) = L$. Then draw a labeled picture that illustrates the definition.

   Solution. For every $\epsilon > 0$ there exists $\delta > 0$ such that if $0 < |x-a| < \delta$, then $|f(x)-L| < \epsilon$. (Sketch: horizontal band $L \pm \epsilon$ and vertical band $a \pm \delta$.)

8. **Problem.** Give an example of a function with a removable discontinuity at $x = 1$ and explain why it is removable.

   Solution. $f(x) = \frac{x^2-1}{x-1}$ simplifies to $x+1$ for $x \neq 1$. Defining $f(1) = 2$ removes the hole.

9. **Problem.** Explain why the two-sided limit fails to exist if the one-sided limits are different.

   Solution. A two-sided limit requires both one-sided limits exist and are equal.

10. **Problem.** For $f(x) = \begin{cases} 2, & x < 1 \\ 5, & x \geq 1 \end{cases}$, find $\lim_{x\to 1^-} f(x)$ and $\lim_{x\to 1^+} f(x)$ and conclude about $\lim_{x\to 1} f(x)$.

    Solution. Left limit 2, right limit 5, so the two-sided limit does not exist.

11. **Problem.** Consider $g(x) = \frac{1}{(x-2)^2}$. Describe the vertical asymptote and explain what $\lim_{x\to 2} g(x)$ tells you.

    Solution. Vertical asymptote at $x = 2$. The limit is $+\infty$, so the graph blows up upward near $x = 2$.

12. **Problem.** Let $h(x) = \frac{x^2-1}{x-1}$ for $x \neq 1$. (a) Find $\lim_{x\to 1} h(x)$. (b) Define $h(1)$ so that h is continuous at 1. (c) Define $h(1)$ in a different way so the limit still exists but h is not continuous at 1. Explain the difference.

Solution. (a) $h(x) = x+1$ for $x \neq 1$, so the limit is 2. (b) Set $h(1) = 2$ to make it continuous. (c) Set $h(1) \neq 2$ (e.g. 0); the limit remains 2 but continuity fails because $h(1) \neq \lim_{x\to 1} h(x)$.