Homework 04 – Solutions

Math 140-002: Calculus I (Spring 2026)
Week 4

Relevant topics: tangent and secant lines; limit definition of the derivative; basic derivative rules; product and quotient rules; trigonometric derivatives; higher-order derivatives

1. **Problem.** Evaluate $\frac{d}{dx}\big(5x^6 - 2x^4 + 7x - 9\big)$.

   Solution.
   $$\frac{d}{dx}\big(5x^6 - 2x^4 + 7x - 9\big) = 30x^5 - 8x^3 + 7.$$

2. **Problem.** Evaluate $\frac{d}{dx}\big(3\sin(x) - 2\cos(x) + x^2\big)$.

   Solution.
   $$\frac{d}{dx}\big(3\sin(x) - 2\cos(x) + x^2\big) = 3\cos(x) + 2\sin(x) + 2x.$$

3. **Problem.** Evaluate $\frac{d}{dx}\big((2x-5)(x^2+1)\big)$.

   Solution.
   $$\frac{d}{dx}\big((2x-5)(x^2+1)\big) = 2(x^2+1) + (2x-5)(2x) = 2x^2 + 2 + 4x^2 - 10x = 6x^2 - 10x + 2.$$

4. **Problem.** Evaluate $\frac{d}{dx}\left(\frac{x^2-3x+1}{x-2}\right)$.

   Solution. Quotient rule:
   $$\frac{d}{dx}\left(\frac{x^2-3x+1}{x-2}\right) = \frac{(2x-3)(x-2) - (x^2-3x+1)(1)}{(x-2)^2}.$$

   Simplify the numerator:
   $$(2x-3)(x-2) - (x^2-3x+1) = (2x^2 - 7x + 6) - (x^2 - 3x + 1) = x^2 - 4x + 5.$$

   So
   $$\left(\frac{x^2-3x+1}{x-2}\right)' = \frac{x^2-4x+5}{(x-2)^2}.$$

5. **Problem.** Evaluate $\frac{d}{dx}\left(\frac{\sin(x)}{x^2+4}\right)$.

   Solution. Quotient rule:
   $$\left(\frac{\sin(x)}{x^2+4}\right)' = \frac{\cos(x)(x^2+4) - \sin(x)(2x)}{(x^2+4)^2}.$$

6. **Problem.** Let $f(x) = x^3 - 6x$. Compute $f''(x)$.

   Solution.
   $$f'(x) = 3x^2 - 6, \qquad f''(x) = 6x.$$

7. **Problem.** Let $f(x) = x^3$. (a) Find the slope of the secant line on $[1, 1+h]$. (b) Compute $\lim_{h\to 0}$ and interpret.

   Solution.
   $$m_{\text{sec}} = \frac{f(1+h) - f(1)}{(1+h) - 1} = \frac{(1+h)^3 - 1}{h}.$$
   Expand $(1+h)^3 = 1 + 3h + 3h^2 + h^3$:
   $$m_{\text{sec}} = \frac{3h + 3h^2 + h^3}{h} = 3 + 3h + h^2.$$
   Then
   $$\lim_{h\to 0} m_{\text{sec}} = \lim_{h\to 0}(3 + 3h + h^2) = 3.$$
   Interpretation: the slope of the tangent line to $y = x^3$ at $x = 1$ is 3.

8. **Problem.** Use the limit definition to compute $f'(0)$ if $f(x) = \sqrt{x+1}$.

   Solution.
   $$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{\sqrt{1+h} - 1}{h}.$$
   Multiply by the conjugate:
   $$f'(0) = \lim_{h\to 0} \frac{(\sqrt{1+h} - 1)(\sqrt{1+h} + 1)}{h(\sqrt{1+h} + 1)} = \lim_{h\to 0} \frac{h}{h(\sqrt{1+h} + 1)} = \lim_{h\to 0} \frac{1}{\sqrt{1+h} + 1} = \frac{1}{2}.$$

9. **Problem.** Consider $f(x) = |x - 2|$. (a) Show f is continuous at 2. (b) Compute one-sided derivatives at 2 and conclude.

   Solution. (a) For $x \neq 2$, $|x - 2| \geq 0$ and $\lim_{x\to 2} |x - 2| = 0 = f(2)$, so f is continuous at 2.

   (b) Using the definition:
   $$f'(2^+) = \lim_{h\to 0^+} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0^+} \frac{|h| - 0}{h} = \lim_{h\to 0^+} \frac{h}{h} = 1.$$
   $$f'(2^-) = \lim_{h\to 0^-} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0^-} \frac{|h|}{h} = \lim_{h\to 0^-} \frac{-h}{h} = -1.$$
   Since $f'(2^-) \neq f'(2^+)$, f is not differentiable at $x = 2$.

10. **Problem.** Find the equation of the tangent line to $y = \cos(x)$ at $x = \pi/3$.

    Solution.
    $$y' = -\sin(x) \quad \Rightarrow \quad m = y'(\pi/3) = -\sin(\pi/3) = -\frac{\sqrt{3}}{2}.$$
    Point: $(\pi/3, \cos(\pi/3)) = \left(\pi/3, \frac{1}{2}\right)$. Tangent line:
    $$y - \frac{1}{2} = -\frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right).$$

11. **Problem.** Let $g(x) = x^4 - 4x^2$. (a) Compute $g'(x), g''(x)$. (b) Horizontal tangents. (c) Concavity.

Solution. (a)

$$g'(x) = 4x^3 - 8x = 4x(x^2 - 2), \qquad g''(x) = 12x^2 - 8.$$

(b) Horizontal tangents: $g'(x) = 0 \Rightarrow 4x(x^2 - 2) = 0$, so

$$x = 0, \quad x = \pm\sqrt{2}.$$

(c) Concavity from $g''(x) = 12x^2 - 8 = 4(3x^2 - 2)$.

$$g''(x) > 0 \Leftrightarrow x^2 > \frac{2}{3} \Leftrightarrow |x| > \sqrt{\frac{2}{3}}.$$

So g is concave up on $(-\infty, -\sqrt{2/3}) \cup (\sqrt{2/3}, \infty)$ and concave down on $(-\sqrt{2/3}, \sqrt{2/3})$.

12. **Problem.** Use a tangent line to estimate $\sqrt{16.4}$ and determine over/under.

Solution. Let $f(x) = \sqrt{x}$ and linearize at $a = 16$.

$$f(16) = 4, \qquad f'(x) = \frac{1}{2\sqrt{x}}, \qquad f'(16) = \frac{1}{8}.$$

Linear approximation:

$$\sqrt{16.4} \approx f(16) + f'(16)(0.4) = 4 + \frac{1}{8}(0.4) = 4 + 0.05 = 4.05.$$

Since $f''(x) = -\frac{1}{4}x^{-3/2} < 0$ for $x > 0$, f is concave down, so the tangent line lies above the curve. Therefore 4.05 is an *overestimate*.