Homework 05 – Solutions

Math 140-002: Calculus I (Spring 2026)
Week 5

Relevant topics: chain rule; implicit differentiation; exponential/logarithmic derivatives; inverse trigonometric derivatives

1. **Problem.** Evaluate $\frac{d}{dx}\left((3x^2+1)^4\right)$.

   Solution.
   $$\frac{d}{dx}\left((3x^2+1)^4\right) = 4(3x^2+1)^3 \cdot 6x = 24x(3x^2+1)^3.$$

2. **Problem.** Evaluate $\frac{d}{dx}\left(e^{2x-1}\right)$.

   Solution.
   $$\frac{d}{dx}\left(e^{2x-1}\right) = e^{2x-1} \cdot 2 = 2e^{2x-1}.$$

3. **Problem.** Evaluate $\frac{d}{dx}\left(\ln(x^2+4x+5)\right)$.

   Solution.
   $$\frac{d}{dx}\left(\ln(x^2+4x+5)\right) = \frac{2x+4}{x^2+4x+5}.$$

4. **Problem.** Evaluate $\frac{d}{dx}(2^x)$.

   Solution.
   $$\frac{d}{dx}(2^x) = 2^x \ln(2).$$

5. **Problem.** Evaluate $\frac{d}{dx}(\arctan(3x))$.

   Solution.
   $$\frac{d}{dx}(\arctan(3x)) = \frac{1}{1+(3x)^2} \cdot 3 = \frac{3}{1+9x^2}.$$

6. **Problem.** Use implicit differentiation to find $\frac{dy}{dx}$ for $x^2+xy+y^2=1$.

   Solution. Differentiate:
   $$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0.$$
   Solve:
   $$(x+2y)\frac{dy}{dx} = -(2x+y) \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{2x+y}{x+2y}.$$

7. **Problem.** For $x^2+xy+y^2=3$, find $\frac{dy}{dx}$ and the tangent line at $(1,1)$.

   Solution. Differentiate:
   $$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{2x+y}{x+2y}.$$

At $(1,1)$:

$$m = -\frac{2(1)+1}{1+2(1)} = -\frac{3}{3} = -1.$$

Tangent line:

$$y - 1 = -1(x-1) \quad \Rightarrow \quad y = -x + 2.$$

8. **Problem.** Differentiate $f(x) = \dfrac{(x^2+1)e^x}{\sqrt{x+1}}$.

   Solution. Write $f(x) = (x^2+1)e^x(x+1)^{-1/2}$. Let $u(x) = (x^2+1)e^x$ and $v(x) = (x+1)^{-1/2}$. Then

$$u'(x) = 2xe^x + (x^2+1)e^x = e^x(x^2+2x+1) = e^x(x+1)^2, \qquad v'(x) = -\frac{1}{2}(x+1)^{-3/2}.$$

   So

$$f'(x) = u'v + uv' = e^x(x+1)^2(x+1)^{-1/2} - (x^2+1)e^x\frac{1}{2}(x+1)^{-3/2}.$$

   Factor $e^x(x+1)^{-3/2}$:

$$f'(x) = e^x(x+1)^{-3/2}\left((x+1)^2(x+1) - \frac{1}{2}(x^2+1)\right) = e^x(x+1)^{-3/2}\left((x+1)^3 - \frac{1}{2}(x^2+1)\right).$$

   Equivalently,

$$f'(x) = \frac{e^x\left(2(x+1)^3 - (x^2+1)\right)}{2(x+1)^{3/2}}.$$

9. **Problem.** Differentiate $y = \arcsin\left(\dfrac{x}{\sqrt{1+x^2}}\right)$.

   Solution. Let $u(x) = \dfrac{x}{\sqrt{1+x^2}} = x(1+x^2)^{-1/2}$. Then

$$u'(x) = (1+x^2)^{-1/2} + x\left(-\frac{1}{2}\right)(1+x^2)^{-3/2}\cdot 2x = (1+x^2)^{-1/2} - x^2(1+x^2)^{-3/2} = \frac{1}{(1+x^2)^{3/2}}.$$

   Also,

$$1 - u(x)^2 = 1 - \frac{x^2}{1+x^2} = \frac{1}{1+x^2} \quad \Rightarrow \quad \sqrt{1-u^2} = \frac{1}{\sqrt{1+x^2}}.$$

   Thus,

$$y' = \frac{u'(x)}{\sqrt{1-u(x)^2}} = \frac{1/(1+x^2)^{3/2}}{1/\sqrt{1+x^2}} = \frac{1}{1+x^2}.$$

10. **Problem.** For $x^2 + y^2 = 4$, find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

    Solution. Differentiate:

$$2x + 2y\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{x}{y}.$$

    Differentiate again:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(-\frac{x}{y}\right) = -\frac{y - x\frac{dy}{dx}}{y^2}.$$

Substitute $\frac{dy}{dx} = -\frac{x}{y}$:

$$\frac{d^2y}{dx^2} = -\frac{y - x\left(-\frac{x}{y}\right)}{y^2} = -\frac{y + \frac{x^2}{y}}{y^2} = -\frac{x^2 + y^2}{y^3}.$$

Since $x^2 + y^2 = 4$ on the curve,

$$\frac{d^2y}{dx^2} = -\frac{4}{y^3}.$$

11. **Problem.** Use implicit differentiation to find $\dfrac{dy}{dx}$ for $xe^y + y = 1$.

    Solution. Differentiate:

    $$\frac{d}{dx}(xe^y) + \frac{d}{dx}(y) = 0.$$

    Product rule and chain rule:

    $$e^y + xe^y \frac{dy}{dx} + \frac{dy}{dx} = 0.$$

    Solve:

    $$(xe^y + 1)\frac{dy}{dx} = -e^y \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{e^y}{xe^y + 1}.$$

12. **Problem.** Prove $\dfrac{d}{dx}\big(\ln(x)\big) = \dfrac{1}{x}$ for $x > 0$.

    Solution. Start from $e^{\ln(x)} = x$ for $x > 0$. Differentiate both sides:

    $$\frac{d}{dx}\left(e^{\ln(x)}\right) = \frac{d}{dx}(x).$$

    Chain rule:

    $$e^{\ln(x)} \cdot \frac{d}{dx}\big(\ln(x)\big) = 1.$$

    Since $e^{\ln(x)} = x$,

    $$x \cdot \frac{d}{dx}\big(\ln(x)\big) = 1 \quad \Rightarrow \quad \frac{d}{dx}\big(\ln(x)\big) = \frac{1}{x}.$$