

Homework 07

Math 140-002: Calculus I (Spring 2026)

Week 7

Relevant topics: related rates; linear approximations and differentials; L'Hôpital's Rule; Rolle's Theorem and the Mean Value Theorem; First Derivative Test

Due: Wednesday, March 4, 2026.

Instructions: Show your work clearly. Problems 1–6 emphasize computational fluency; Problems 7–12 emphasize concepts, explanation, and multi-step processes.

1. Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{e^{5x} - 1}{x}.$$

2. Use linear approximation to estimate $\sqrt{25.3}$.
3. Let $V = \frac{4}{3}\pi r^3$ be the volume of a sphere. Use differentials to estimate the change in volume when the radius changes from $r = 4$ cm to $r = 4.05$ cm.
4. A circle is expanding so that its radius is increasing at a rate of $\frac{dr}{dt} = 2$ cm/s. Find $\frac{dA}{dt}$ when $r = 7$ cm, where A is the area of the circle.
5. Let $f(x) = x^3 - 3x$ on the interval $[-2, 2]$.

(a) Verify that the hypotheses of the Mean Value Theorem apply on $[-2, 2]$.

(b) Find all values $c \in (-2, 2)$ such that

$$f'(c) = \frac{f(2) - f(-2)}{2 - (-2)}.$$

6. Use the First Derivative Test to find all local extrema of

$$f(x) = x^{2/3}(x - 4).$$

(Your answer should include the critical numbers, a sign chart or sign analysis for $f'(x)$, and the classification of each critical number.)

7. (Related rates) A 15-ft ladder is leaning against a vertical wall. The bottom of the ladder is sliding away from the wall at a rate of $\frac{dx}{dt} = 1.5$ ft/s. How fast is the top of the ladder sliding down the wall when the bottom is 9 ft from the wall?

8. (L'Hôpital; indeterminate form) Evaluate the limit:

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 4x} - x).$$

9. (L'Hôpital; indeterminate powers) Evaluate the limit:

$$\lim_{x \rightarrow 0^+} (1 + 2x)^{3/x}.$$

(Hint: Let $y = (1 + 2x)^{3/x}$ and take \ln of both sides.)

10. (MVT; concept check) Prove that for all real x ,

$$|\sin x| \leq |x|.$$

(Hint: Apply the Mean Value Theorem to $f(t) = \sin t$ on an interval with endpoints 0 and x .)

11. (Rolle's Theorem; multistep) Consider the function

$$g(x) = x^5 - 5x + 1.$$

(a) Show that $g(x)$ has at most three real zeros.

(b) Show that $g(x)$ has exactly one real zero.

12. (First Derivative Test; written explanation) Suppose f is differentiable on \mathbb{R} and $f'(x) > 0$ for all x .

(a) Explain why f is one-to-one.

(b) Conclude that the equation $f(x) = k$ has at most one real solution for any constant k .