Math 140: Calculus I (Spring 2026)
Homework Week 10

Relevant topics: first derivative test; second derivative test; local and absolute extrema; limits at infinity; curve sketching; optimization.

Instructions: Show your work clearly. Problems 1–6 emphasize computational fluency. Problems 7–12 emphasize conceptual reasoning, curve sketching, and applications.

1. Use the First Derivative Test to find and classify all local extrema of

$$f(x) = x^3 - 6x^2 + 9x.$$

   Your answer should include the critical numbers, a sign chart for $f'(x)$, and the classification of each critical number.

2. Use the Second Derivative Test to find and classify all local extrema of

$$f(x) = x^4 - 4x^2 + 3.$$

3. Find the absolute maximum and absolute minimum values of

$$f(x) = x^3 - 3x + 1$$

   on the closed interval $[-2, 2]$.

4. Evaluate the limit

$$\lim_{x \to \infty} \frac{3x^2 + 5x - 2}{2x^2 - x + 1}.$$

5. Evaluate the limit using L'Hôpital's Rule

$$\lim_{x\to\infty} \frac{xe^x}{e^x+2}.$$

6. Evaluate the limit

$$\lim_{x\to\infty} \left(x - \ln(x+1)\right).$$

Determine whether the limit diverges to ∞, $-\infty$, or approaches a finite value.

7. Use the curve sketching checklist to analyze the function

$$f(x) = \frac{x}{x^2+4}.$$

Your analysis should include

- intercepts
- horizontal and vertical asymptotes
- critical numbers
- intervals of increase/decrease
- concavity and inflection points

Use this information to sketch the graph of $f(x)$.

8. Use the curve sketching checklist to analyze the function

$$f(x) = xe^{-x}.$$

Include limits at infinity, critical numbers, intervals of increase/decrease, and concavity.

9. A rectangular garden is to be built along a wall using 120 meters of fencing for the other three sides.

   Find the dimensions that maximize the area of the garden.

10. A box with a square base and no top must have volume 500 cm^3.

    Find the dimensions of the box that minimize the amount of material used.

11. A point on the curve

$$y = x^2 + 1$$

    is connected to the origin by a line segment.

    Find the point on the curve that minimizes the distance to the origin.

12. Use the curve sketching checklist to analyze the function

$$f(x) = \ln(x) - \frac{x}{2}.$$

    Your analysis should include

    - domain
    - intercepts
    - limits at infinity
    - critical numbers
    - increasing/decreasing intervals
    - concavity and inflection points

    Use this information to sketch the graph.