Quiz 05 – Solutions

1. Let $u(x) = 2\sin(x) - 3\cos(x)$ and $v(x) = x^2 + 1$. Then $u'(x) = 2\cos(x) + 3\sin(x)$ and $v'(x) = 2x$.

$$f'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{(v(x))^2} = \frac{\big(2\cos(x) + 3\sin(x)\big)(x^2+1) - \big(2\sin(x) - 3\cos(x)\big)(2x)}{(x^2+1)^2}.$$

2.
$$g(x) = x^2\cos(x)$$

First derivative:
$$g'(x) = 2x\cos(x) - x^2\sin(x).$$

Second derivative:

$$g''(x) = 2\cos(x) - 2x\sin(x) - 2x\sin(x) - x^2\cos(x) = 2\cos(x) - 4x\sin(x) - x^2\cos(x).$$

3.
$$y = \sin(3x^2) - 4\cos(x)$$
$$y' = \cos(3x^2)\cdot 6x + 4\sin(x) = 6x\cos(3x^2) + 4\sin(x).$$

4. Differentiate both sides:
$$2x + x\frac{dy}{dx} + y = 0.$$

Solve for $\dfrac{dy}{dx}$:
$$x\frac{dy}{dx} = -(2x+y) \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{2x+y}{x}.$$