Quiz 08 – Solutions

1. Use $f(x) = \sqrt{x}$ and linearize at $a = 9$, since $\sqrt{9} = 3$.

$$f'(x) = \frac{1}{2\sqrt{x}}, \qquad f'(9) = \frac{1}{6}.$$

Thus the linearization at $x = 9$ is

$$L(x) = f(9) + f'(9)(x - 9) = 3 + \frac{1}{6}(x - 9).$$

Now approximate:

$$\sqrt{9.2} \approx L(9.2) = 3 + \frac{1}{6}(0.2) = 3 + \frac{1}{30} = \frac{91}{30} \approx 3.0333.$$

Therefore,

$$\boxed{\sqrt{9.2} \approx 3.0333.}$$

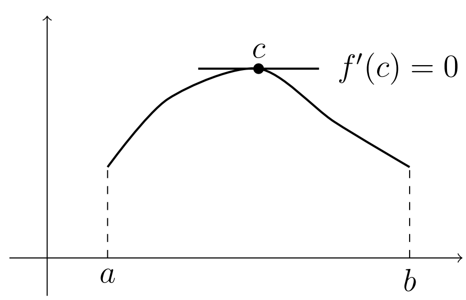
2. (a) **Rolle's Theorem:** If f is continuous on $[a, b]$, differentiable on (a, b), and

$$f(a) = f(b),$$

then there exists at least one number $c \in (a, b)$ such that

$$f'(c) = 0.$$

(b) A sketch illustrating Rolle's Theorem is shown below.

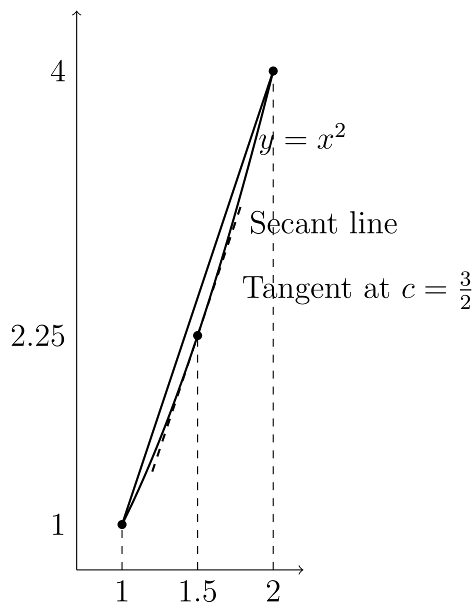


(c) Geometrically, if a smooth curve begins and ends at the same height, then at some interior point c the tangent line must be horizontal.

3. (a) **Mean Value Theorem:** If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists at least one number $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

(b) A sketch illustrating the Mean Value Theorem is shown below.



(c) Geometrically, the Mean Value Theorem says that the slope of the secant line over $[a, b]$ equals the slope of the tangent line at some interior point c.

4. Let

$$f(x) = x^3 - 6x^2 + 9x.$$

(a) Critical numbers occur where $f'(x) = 0$ or where $f'(x)$ is undefined.

$$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x - 1)(x - 3).$$

Thus

$$f'(x) = 0 \quad \Rightarrow \quad x = 1, 3.$$

So the critical numbers are

$$\boxed{x = 1 \text{ and } x = 3.}$$

(b) Use the First Derivative Test. Since

$$f'(x) = 3(x - 1)(x - 3),$$

we check the sign of $f'(x)$ on the intervals determined by $x = 1$ and $x = 3$:

x	$(-\infty, 1)$	$(1, 3)$	$(3, \infty)$
$f'(x)$	$+$	$-$	$+$

So f is increasing on $(-\infty, 1)$, decreasing on $(1, 3)$, and increasing on $(3, \infty)$. Therefore:

- At $x = 1$, $f'(x)$ changes from positive to negative, so f has a local maximum at $x = 1$.

- At $x = 3$, $f'(x)$ changes from negative to positive, so f has a local minimum at $x = 3$.

Now evaluate:

$$f(1) = 1 - 6 + 9 = 4, \qquad f(3) = 27 - 54 + 27 = 0.$$

Hence the local maximum is at

$$\boxed{(1, 4)}$$

and the local minimum is at

$$\boxed{(3, 0).}$$