Quiz: Approximation Sums and Definite Integrals Solutions

Math 140: Calculus I

1. Use right-endpoint rectangles to evaluate the definite integral

$$\int_{-1}^{2} x\,dx$$

as the limit of a sequence of approximation sums.

(a) Divide the interval $[-1, 2]$ into n equal subintervals. Find Δx and the right-endpoint x_i for the ith subinterval.

The interval $[-1, 2]$ has length

$$2 - (-1) = 3.$$

Thus,

$$\Delta x = \frac{3}{n}.$$

Using right endpoints, the ith right-endpoint is

$$x_i = a + i\Delta x = -1 + i\left(\frac{3}{n}\right).$$

Therefore,

$$\boxed{\Delta x = \frac{3}{n}, \qquad x_i = -1 + \frac{3i}{n}.}$$

(b) Write a summation formula for the right-endpoint approximation A_n.

Since $f(x) = x$, the right-endpoint approximation is

$$A_n = \sum_{i=1}^{n} f(x_i)\Delta x.$$

Substituting $f(x) = x$, $x_i = -1 + \frac{3i}{n}$, and $\Delta x = \frac{3}{n}$, we get

$$A_n = \sum_{i=1}^{n} \left(-1 + \frac{3i}{n}\right)\frac{3}{n}.$$

Thus,

$$\boxed{A_n = \sum_{i=1}^{n} \left(-1 + \frac{3i}{n}\right)\frac{3}{n}.}$$

(c) Replace the summation with a formula (do not leave sigma notation). Simplify your expression for A_n.

Distribute $\frac{3}{n}$:

$$A_n = \sum_{i=1}^{n} \left(-\frac{3}{n} + \frac{9i}{n^2} \right).$$

Now split the sum:

$$A_n = \sum_{i=1}^{n} \left(-\frac{3}{n} \right) + \sum_{i=1}^{n} \frac{9i}{n^2}.$$

So,

$$A_n = -\frac{3}{n} \sum_{i=1}^{n} 1 + \frac{9}{n^2} \sum_{i=1}^{n} i.$$

Using

$$\sum_{i=1}^{n} 1 = n \qquad \text{and} \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2},$$

we obtain

$$A_n = -\frac{3}{n}(n) + \frac{9}{n^2} \cdot \frac{n(n+1)}{2}.$$

Simplifying,

$$A_n = -3 + \frac{9(n+1)}{2n}.$$

Combine into a single fraction:

$$A_n = \frac{-6n + 9n + 9}{2n} = \frac{3n+9}{2n}.$$

Hence,

$$\boxed{A_n = \frac{3n+9}{2n} = \frac{3}{2} + \frac{9}{2n}.}$$

(d) Take the limit as $n \to \infty$ to evaluate the definite integral.

Now compute

$$\lim_{n\to\infty} A_n = \lim_{n\to\infty} \left(\frac{3}{2} + \frac{9}{2n} \right).$$

Since

$$\lim_{n\to\infty} \frac{9}{2n} = 0,$$

we get

$$\lim_{n\to\infty} A_n = \frac{3}{2}.$$

Therefore,

$$\boxed{\int_{-1}^{2} x \, dx = \frac{3}{2}.}$$