

# Derivatives Quiz

Math 140: Calculus with Analytic Geometry

## Answer Key

### 1. The limit definition of the derivative.

Let

$$f(x) = x^2 - 3x.$$

Use the limit definition

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

to find  $f'(x)$ . Your work must show the difference quotient and its simplification before the limit is evaluated.

**Solution.** Using the limit definition,

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 3h}{h} \\ &= \lim_{h \rightarrow 0} (2x + h - 3) \\ &= 2x - 3. \end{aligned}$$

The cancellation of  $h$  occurs before substituting  $h = 0$ .

## 2. Linearity, product rule, and quotient rule.

Differentiate as indicated. Assume  $x > 0$  where a radical or fractional exponent appears, and assume  $x \neq 1$  for  $q$ . You do not need to expand or combine answers obtained from the product or quotient rule.

(a) Find  $p'(x)$  if  $p(x) = (\sqrt{x} + 1)(x^2 - 2x)$ .

(b) Find  $q'(x)$  if  $q(x) = \frac{x^2 + 1}{x - 1}$ .

(c) Find  $r''(x)$  if  $r(x) = x^{5/2} - 3x^{3/2}$ .

**Solution.** For part (a), the product rule gives

$$p'(x) = \frac{1}{2\sqrt{x}}(x^2 - 2x) + (\sqrt{x} + 1)(2x - 2).$$

For part (b), the quotient rule gives

$$q'(x) = \frac{(x - 1)(2x) - (x^2 + 1)(1)}{(x - 1)^2}.$$

For part (c), differentiate twice:

$$r'(x) = \frac{5}{2}x^{3/2} - \frac{9}{2}x^{1/2}, \quad r''(x) = \frac{15}{4}x^{1/2} - \frac{9}{4}x^{-1/2}.$$

The product-rule and quotient-rule expressions may be simplified, but simplification is not required.

3. A tangent line involving a trigonometric function.

Let

$$g(x) = x \cos x.$$

Find an equation of the line tangent to the graph of  $g$  at  $x = \frac{\pi}{2}$ . Give your answer in point-slope form.

**Solution.** The point on the graph is

$$\left(\frac{\pi}{2}, g\left(\frac{\pi}{2}\right)\right) = \left(\frac{\pi}{2}, 0\right).$$

Using the product rule,

$$g'(x) = \cos x - x \sin x.$$

Thus the tangent slope is

$$g'\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) - \frac{\pi}{2} \sin\left(\frac{\pi}{2}\right) = -\frac{\pi}{2}.$$

The tangent line is

$$y - 0 = -\frac{\pi}{2} \left(x - \frac{\pi}{2}\right).$$