

Homework 1

Math 140: Calculus with Analytic Geometry

Name: _____

Due date: _____

Directions. Show enough work to support each answer. Give exact values unless a decimal approximation is requested. No calculators or other electronic devices are allowed.

1. Polynomial equations.

Solve each equation over the real numbers.

(a) $2x^3 + x^2 - 8x - 4 = 0$

(b) $3x^2 - 5x - 4 = 0$

2. Rational expressions and graph features.

Let

$$r(x) = \frac{x^2 - 5x + 6}{x^2 - x - 6}.$$

- (a) Factor and simplify $r(x)$, retaining all restrictions from the original expression.
- (b) State the domain of r in interval notation.
- (c) Identify any holes and all vertical and horizontal asymptotes of the graph.

3. Composition and domain.

Let

$$f(x) = \frac{1}{x+1} \quad \text{and} \quad g(x) = \sqrt{4-x}.$$

Find formulas for $(f \circ g)(x)$ and $(g \circ f)(x)$. State the domain of each composite function in interval notation.

4. Inverse functions.

Let $h(x) = (3x + 5)/(2x - 1)$.

- (a) State the domain of h .
- (b) Find $h^{-1}(x)$ and state its domain.
- (c) Verify your formula by showing that $h^{-1}(h(x)) = x$.

5. Exponential functions and models.

- (a) For $p(x) = -3 \cdot 2^{x-1} + 5$, describe the transformations of $y = 2^x$ and state the domain, range, and horizontal asymptote of p .
- (b) A quantity is modeled by $Q(t) = 240e^{kt}$ and doubles every 6 years. Find the exact value of k and then find $Q(15)$ in exact form.

6. Logarithms and exponential equations.

- (a) Condense $2 \ln(x + 1) - \frac{1}{2} \ln x - \ln 3$ into one logarithm, assuming $x > 0$.
- (b) Solve $5^{x+1} = 3^{2x}$, expressing the answer using logarithms.
- (c) Solve $\ln(x - 2) + \ln x = \ln(3x + 4)$, checking all domain restrictions.

7. Radian measure and arc length.

- (a) Convert -315° to radians and give a coterminal angle in $[0, 2\pi)$.
- (b) A wheel with radius 14 centimeters rotates through $5\pi/7$ radians. Find the exact distance traveled by a point on its rim.

8. Unit-circle trigonometry.

Let $\theta = 4\pi/3$.

- (a) Identify the quadrant and reference angle of θ .
- (b) Find the exact values of all six trigonometric functions at θ .
- (c) Find an angle in $[0, 2\pi)$ that is coterminal with $-2\pi/3$.

9. Trigonometric identities and equations.

- (a) If $\cos \theta = -5/13$ and θ lies in Quadrant II, find the exact values of $\sin \theta$ and $\tan \theta$.
- (b) Solve $2 \sin^2 x - \sin x = 0$ on $[0, 2\pi)$.

10. Graphs and inverse trigonometric functions.

- (a) For $y = 3 \sin(2(x + \pi/4)) - 1$, state the amplitude, period, horizontal shift, midline, and range.
- (b) Evaluate exactly: $\arccos(-\sqrt{3}/2)$, $\arctan(\tan(3\pi/4))$, and $\arcsin(\sin(5\pi/6))$.
- (c) Simplify $\sin(\arccos x)$ for $-1 \leq x \leq 1$.