

# Homework 2

## Math 140: Calculus with Analytic Geometry

Name: \_\_\_\_\_

Due date: \_\_\_\_\_

**Directions.** Show enough work to support each answer. Give exact values. Name any limit law or theorem that is essential to your reasoning. No calculators or other electronic devices are allowed.

### 1. Algebraic techniques for indeterminate limits.

Evaluate each limit. Show the algebraic step that removes the indeterminate form.

(a)  $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 9}$

(b)  $\lim_{x \rightarrow 0} \frac{\sqrt{9+x} - 3}{x}$

(c)  $\lim_{x \rightarrow 2} \left( \frac{1}{x-2} - \frac{4}{x^2-4} \right)$

### 2. Limits of transcendental functions.

Evaluate each limit. Angles in trigonometric limits are measured in radians.

(a)  $\lim_{x \rightarrow 0} \frac{\sin(5x)}{3x}$

(b)  $\lim_{x \rightarrow 1} \frac{\sin(x^2 - 1)}{x - 1}$

(c)  $\lim_{x \rightarrow 1} \ln \left( \frac{x^2 - 1}{x - 1} \right)$

### 3. The Squeeze Theorem.

(a) Use the Squeeze Theorem to evaluate

$$\lim_{x \rightarrow 0} x \sin \left( \frac{1}{x} \right).$$

Your solution must include explicit lower and upper bounds.

(b) For  $0 < x < 1$ , suppose the following inequalities are known:

$$x - \frac{x^3}{6} \leq \sin x \leq x - \frac{x^3}{6} + \frac{x^5}{120}.$$

Use these inequalities and the Squeeze Theorem to evaluate

$$\lim_{x \rightarrow 0^+} \frac{x - \sin x}{x^3}.$$

Show clearly how you obtain the lower and upper bounds for the quotient.

4. **Making a piecewise function continuous.**

Find the values of  $a$  and  $b$  that make

$$f(x) = \begin{cases} x + a, & x < 0, \\ x^2 + b, & 0 \leq x < 2, \\ 3x - 1, & x \geq 2 \end{cases}$$

continuous for every real number. Justify your answer by checking continuity at both transition points.

5. **The Intermediate Value Theorem.**

Use the Intermediate Value Theorem to prove that the equation

$$\cos x = x$$

has at least one solution in the interval  $(0, 1)$ . Clearly identify the function to which you apply the theorem and verify every hypothesis. Does the Intermediate Value Theorem prove that the solution is unique? Explain.