

Homework 3

Math 140: Calculus with Analytic Geometry

Name: _____

Due date: _____

Directions. Show enough work to support each answer. Give exact values. When differentiating, state the rule or rules you use. Do not use derivative rules in Problems 1–3. No calculators or other electronic devices are allowed.

1. Secant lines and average rate of change.

Let $f(x) = x^2 - 2x$.

- (a) Find the average rate of change of f from $x = -1$ to $x = 2$.
- (b) Write an equation of the secant line through the corresponding points on the graph.

2. The derivative at a point.

Let $f(x) = 1/(x + 1)$.

- (a) Simplify $\frac{f(1+h) - f(1)}{h}$ for $h \neq 0$.
- (b) Use the limit definition to find $f'(1)$.
- (c) Find an equation of the tangent line to $y = f(x)$ at $x = 1$.

3. The derivative as a function.

Starting from

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h},$$

find the derivative function for $f(x) = x^2 - 3x + 1$. Simplify the difference quotient before taking the limit.

4. Where derivatives do and do not exist.

For each function and specified input, determine whether the function is continuous and whether it is differentiable. Justify your conclusions using appropriate limits. Identify any vertical tangent.

- (a) $f(x) = |x - 2|$ at $x = 2$.
- (b) $g(x) = x^{2/3}$ at $x = 0$.
- (c) $p(x) = \frac{x^2 - 1}{x - 1}$ at $x = 1$.

5. Linearity and the power rule.

Find the derivative of each function. Rewrite radicals and reciprocals as powers before differentiating.

- (a) $f(x) = 4x^5 - 3x^2 + 7x - 9$.
- (b) $g(x) = \sqrt{x} + \frac{3}{\sqrt{x}}$, where $x > 0$.
- (c) $h(x) = 5x^{2/3} - 2x^{-3}$, where $x \neq 0$.

6. The product rule.

Differentiate each function. You do not need to expand your final answer unless expansion makes it simpler.

- (a) $f(x) = (x^2 + 1)(x^3 - 2x)$.
- (b) $g(x) = x^{2/3}(x^2 - 3)$, where $x \neq 0$.
- (c) $p(x) = x(x - 1)(x + 2)$.

7. The quotient rule and simplifying first.

Find the derivative of each function. Preserve any domain restriction from the original formula.

- (a) $f(x) = \frac{x^3 - 2x}{x^2 + 1}$.
- (b) $g(x) = \frac{x^{5/2} + 2x^{3/2}}{\sqrt{x}}$, where $x > 0$. Simplify first.
- (c) $p(x) = \frac{(x - 1)(x^2 + 2)}{x - 1}$, where $x \neq 1$. Simplify first.

8. Finding a tangent line.

Let $f(x) = (x^2 + 1)(x - 1)$.

- (a) Find $f'(x)$ using the product rule.
- (b) Find $f(1)$ and $f'(1)$.
- (c) Find an equation of the tangent line to the graph at $x = 1$.

9. Horizontal tangents and a specified slope.

Let $f(x) = x/(x^2 + 1)$.

- (a) Find $f'(x)$.
- (b) Find all points on the graph where the tangent line is horizontal.
- (c) Find all points on the graph where the tangent line has slope $-1/8$.

10. Reading a function from its derivative.

Let $F(x) = x^2(x - 3)$.

- (a) Find $F'(x)$ and write it in factored form.
- (b) Find all points on the graph of F where the tangent line is horizontal.
- (c) Determine where the graph of F is rising and where it is falling. Justify your answer from the sign of $F'(x)$.