

Homework 4

Math 140: Calculus with Analytic Geometry

Name: _____

Due date: _____

Directions. Show enough work to support each answer. Give exact values and state the differentiation rules you use. Simplify when it makes an answer easier to interpret. No calculators or other electronic devices are allowed.

1. Combining derivative rules.

Differentiate each function. Retain any restrictions on its domain.

(a) $f(x) = \left(x^2 + \frac{1}{x}\right)(x^3 - 2x)$, where $x \neq 0$.

(b) $g(x) = \frac{x^2\sqrt{x+1}}{x-1}$, where $x > -1$ and $x \neq 1$.

2. The chain rule in layers.

Find the derivatives.

(a) $f(x) = \sqrt{1 + (x^2 - 3x)^2}$.

(b) $g(x) = \sin^3(2x) - \cos(3x^2)$.

3. Tangent lines and the chain rule.

Let $f(x) = \frac{x}{(x^2 + 1)^2}$.

(a) Find $f'(x)$ and write it as one fraction.

(b) Find equations of all horizontal tangent lines to the graph.

4. Implicit differentiation on an ellipse.

The curve $x^2 + xy + y^2 = 7$ contains $(1, 2)$.

(a) Find dy/dx at every point where $x + 2y \neq 0$.

(b) Find the tangent line at $(1, 2)$.

(c) Determine all points on the curve where the tangent is horizontal.

5. The folium of Descartes.

Consider the curve $x^3 + y^3 = 6xy$ near $(3, 3)$.

(a) Find an expression for dy/dx wherever the resulting denominator is nonzero.

(b) Find the tangent line at $(3, 3)$.

6. Exponential and logarithmic functions.

Find the derivative of each function on its domain.

(a) $f(x) = e^{x^2} \ln(1 + x^2)$.

(b) $g(x) = \ln\left(\frac{\sqrt{x+2}}{x^2+1}\right)$, where $x > -2$.

(c) $h(x) = x^x$, where $x > 0$. (Take logarithms first.)

7. Deriving the derivative of the natural logarithm.

Use the identity $e^{\ln x} = x$ for $x > 0$.

(a) Differentiate both sides with respect to x and solve for $\frac{d}{dx} \ln x$. Explain why your division is valid.

(b) Use your result to find $\frac{d}{dx} \ln(x^2 + 4)$.

(c) Find the tangent line to $y = \ln(x^2 + 4)$ at $x = 1$.

8. Inverse trigonometric derivatives.

Differentiate the functions; state their derivative domains.

(a) $f(x) = \arcsin(2x - 1)$.

(b) $g(x) = \arctan(x^2) + \arccos(x/2)$.

9. Higher derivatives and concavity.

Let $f(x) = xe^{-x}$.

(a) Find $f'(x)$ and $f''(x)$.

(b) Find where $f''(x) = 0$ and determine the sign of f'' on either side.

(c) Find the equation of the tangent line at $x = 0$.

10. Implicit second derivatives.

For the curve $x^2 + 4y^2 = 20$, work near the point $(2, 2)$, where y is a function of x .

(a) Find y' and y'' by differentiating the equation twice.

(b) Find $y''(2)$ and interpret its sign geometrically for this branch.