Calculus with Analytic Geometry

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1 Key Topics

Today we introduce several classic calculus problems that we will investigate in further detail this semester.

1.1 Slope of a Graph

It is known that the slope of the graph of a linear function $f(x) = mx + b$ is equal to m. However, even the graph of non-linear (continuous) functions must have a slope, for example consider the graph of a function (blue) shown in Figure 1. We say that the slope of the graph of the function (blue) is equal to the slope of the line segment (green) at the point where the two intersect. The green line is known as the tangent line of the function. The slope of the tangent line is also known as the instantaneous rate of change or the derivative of the function.

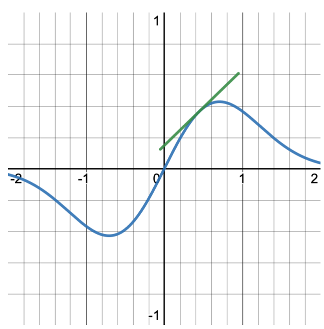

Figure 1: Graph of a function (blue) with tangent line (green)

1.2 Extreme Values

Once we have established the slope of a graph, we can identify extreme values by locating points on the graph where the slope is zero. For example, consider the graph of a function (blue) shown in Figure 2. Note that the green and red tangent lines have a slope of zero. Moreover, the green tangent line corresponds to a point on the graph that is a maximum and the red tangent line corresponds to a point on the graph that is a minimum.

1.3 Area

Another classical problem in calculus is that of finding the area bounded by the graph of a function and the x-axis. For example, consider the area shown in Figure 3. Since the region shown is a triangle with base 2 and height 4, it follows that the area is 4.

We can use the definite integral to calculate the area of more complicated regions, for example consider the area shown in Figure 4, which is equal to $\pi/2$.

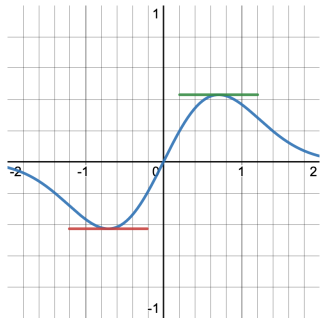

Figure 2: Graph of a function (blue) with tangent lines (green and red) with zero slope

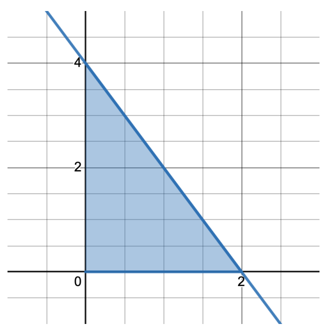

Figure 3: Area bounded by function (blue) and the x-axis over the interval $[0, 2]$

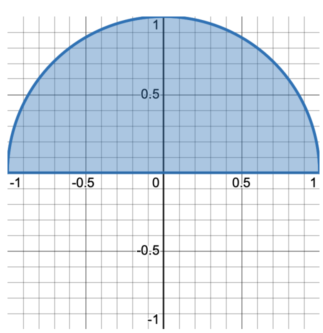

Figure 4: Area bounded by function (blue) and the x-axis over the interval $[-1, 1]$