

Precalculus Review Quiz

Math 140: Calculus with Analytic Geometry

Name: _____

Date: _____

Directions. Show enough work to support each answer. No calculators or other electronic devices are allowed.

1. Rational expressions and domains.

Let

$$r(x) = \frac{x^2 - x - 6}{x^2 - 4x + 3}.$$

- (a) Factor the numerator and denominator and simplify $r(x)$.
- (b) State the domain of the *original* function in interval notation.
- (c) Identify any hole and any vertical asymptote of its graph.

2. Polynomial equations.

Solve each equation over the real numbers.

- (a) $x^3 - 4x^2 - x + 4 = 0$
- (b) $2x^2 + 4x - 3 = 0$

3. Composition and domain.

Let

$$f(x) = \sqrt{x+1} \quad \text{and} \quad g(x) = \frac{1}{x-2}.$$

Find formulas for $(f \circ g)(x)$ and $(g \circ f)(x)$. State the domain of each composite function in interval notation.

4. Inverse functions.

The function

$$h(x) = \frac{2x-3}{x+4}$$

has domain $\mathbb{R} \setminus \{-4\}$.

- (a) Find $h^{-1}(x)$.
- (b) State the domain of h^{-1} .
- (c) Verify your answer by showing that $h(h^{-1}(x)) = x$.

5. Exponential and logarithmic functions.

- (a) Solve $3^{2x-1} = 7$. Express your answer using logarithms.
- (b) Solve $\ln(x-1) + \ln(x+3) = \ln(5x+7)$, checking all domain restrictions.
- (c) Expand $\ln\left(\frac{x^3\sqrt{x+1}}{e^{2x}}\right)$, assuming $x > 0$.

6. Right-triangle trigonometry.

Let x be an acute angle in a right triangle and suppose that

$$\cos(x) = \frac{2}{3}.$$

- (a) Draw a right triangle that represents this information and label the side adjacent to x and the hypotenuse.
- (b) Use the Pythagorean theorem to find the length of the side opposite x .
- (c) Find the exact values of $\sin(x)$ and $\tan(x)$.

7. Unit-circle trigonometry.

Consider the angle $\theta = \frac{5\pi}{6}$.

- (a) Find the exact coordinates of the point of intersection of the ray that forms the angle θ with the unit circle.
- (b) Find the exact values of $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$.
- (c) Find the angle $\alpha \in [0, 2\pi)$ whose ray intersects the unit circle at the point $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

8. Area and perimeter.

A window consists of a rectangle that is 12 feet tall and 8 feet wide with a semicircle attached along the entire 8-foot top edge. The top edge of the rectangle is the diameter of the semicircle.

- (a) Find the radius of the semicircle.
- (b) Find the total area of the window. Give an exact answer in terms of π .
- (c) Find the outside perimeter of the window. Do not include the shared edge between the rectangle and semicircle.