

Precalculus Review Quiz 2

Math 140: Calculus with Analytic Geometry

Answer Key

1. Rational function and domain.

Let

$$r(x) = \frac{x^2 - 5x - 14}{x^2 - x - 6}.$$

- (a) Factor the numerator and denominator and simplify $r(x)$.
- (b) State the domain of the *original* function in interval notation.
- (c) Identify the coordinates of any hole and the equation of any vertical asymptote.

Solution. Factoring gives

$$r(x) = \frac{(x-7)(x+2)}{(x-3)(x+2)} = \frac{x-7}{x-3}, \quad x \neq -2, 3.$$

Thus the domain is

$$(-\infty, -2) \cup (-2, 3) \cup (3, \infty).$$

The canceled factor produces a hole. Using the simplified formula, its y -coordinate is

$$\frac{-2-7}{-2-3} = \frac{9}{5}.$$

Thus the hole is $(-2, \frac{9}{5})$, and the remaining denominator produces the vertical asymptote $x = 3$.

2. Composition of polynomial, logarithmic, and exponential functions.

Let

$$f(x) = \ln x, \quad \text{and} \quad g(x) = \frac{e^{x^2-1}}{e^{x^2+x-7}}.$$

- (a) Find and simplify a formula for $(f \circ g)(x)$. State the domain of $(f \circ g)(x)$ in interval notation.
- (b) Find the x -intercept of the graph of $y = (f \circ g)(x)$.

Solution. For part (a), use the quotient property of logarithms and then the inverse relationship between logarithmic and exponential functions:

$$\begin{aligned} (f \circ g)(x) &= \ln\left(\frac{e^{x^2-1}}{e^{x^2+x-7}}\right) \\ &= \ln(e^{x^2-1}) - \ln(e^{x^2+x-7}) \\ &= (x^2 - 1) - (x^2 + x - 7) \\ &= 6 - x. \end{aligned}$$

For part (b), an x -intercept occurs when the output is zero. Thus

$$6 - x = 0 \implies x = 6.$$

Therefore, the x -intercept is $(6, 0)$.

3. **Right-triangle trigonometry.**

Evaluate each expression exactly. Draw and label a right triangle to support your answers.

$$\sin\left(\arctan\left(\frac{3}{2}\right)\right) \quad \text{and} \quad \cos\left(\arctan\left(\frac{3}{2}\right)\right).$$

Solution. Let $\theta = \arctan(3/2)$. Then $\tan \theta = 3/2$. Since the range of arctangent is $(-\pi/2, \pi/2)$ and the tangent is positive, θ lies in Quadrant I. In a corresponding right triangle, take the opposite side to have length 3 and the adjacent side to have length 2. The hypotenuse is

$$\sqrt{3^2 + 2^2} = \sqrt{13}.$$

Therefore,

$$\sin\left(\arctan\left(\frac{3}{2}\right)\right) = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

and

$$\cos\left(\arctan\left(\frac{3}{2}\right)\right) = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}.$$