

Calculus I Review II Worksheet

Math 140: Calculus with Analytic Geometry

Name: _____

Date: _____

Directions. Show enough work to support each answer. Give exact values. No calculators or other electronic devices are allowed.

1. The limit definition of the derivative.

Use the limit definition to find each derivative as a function of x . Show the algebra that allows you to evaluate the limit. Do not use differentiation rules.

(a) $f(x) = x^2 - 3x + 1$.

(b) $g(x) = \sqrt{x+5}$, for $x > -5$.

2. Linearity, power, product and quotient rules.

In parts (a)–(c), differentiate each function. Show the rules you use and simplify where helpful. Use the product rule in part (b) and the quotient rule in part (c).

(a) $p(x) = 5x^{3/2} - \frac{4}{x^2} + 7$ for $x > 0$.

(b) $q(x) = (x^2 + 2)(x^3 - 3x)$.

(c) $r(x) = \frac{x^2 + 1}{x - 2}$, $x \neq 2$.

(d) For r in part (c), find every point where its tangent line is horizontal.

3. Trigonometric and higher order derivatives.

- (a) Starting from $\tan x = \sin x / \cos x$, use the quotient rule to derive a formula for $\frac{d}{dx}(\tan x)$ wherever $\tan x$ is defined. State the identity used in the simplification.
- (b) If $f(x) = x^2 \sin x$, find $f'(x)$ and $f''(x)$.
- (c) Find the tangent line to the graph of f in part (b) at $x = \pi$.

4. The chain rule with other differentiation rules.

Differentiate each function. For part (c), also find the indicated tangent line.

(a) $F(x) = (1 + 3x^2)^{5/2}$.

(b) $G(x) = \sec(5x)$.

(c) $H(x) = \frac{\cos x}{1 + x^2}$. Find the tangent line at $x = 0$.

(d) Write $\csc x = (\sin x)^{-1}$ and use the chain rule to derive a formula for $\frac{d}{dx}(\csc x)$ wherever it is defined.

5. Implicit differentiation.

Consider the curve $x^2y + y^2 = 12$.

- (a) Verify that $(2, 2)$ lies on the curve. Differentiate implicitly, showing the product rule on x^2y , and solve for y' where possible.
- (b) Find the tangent line at $(2, 2)$.
- (c) Find all points on the curve with a horizontal tangent. Check that your expression for y' has nonzero denominator at each point.

6. Exponential, logarithmic and inverse trigonometric derivatives.

Differentiate in parts (a)–(c). In part (d), derive a new inverse derivative formula.

(a) $A(x) = \ln\left(\frac{x^2 + 1}{\sqrt{4 - x^2}}\right)$, for $-2 < x < 2$.

(b) $B(x) = x^2 e^{\sin(3x)}$.

(c) $C(x) = \arcsin(2x)$. State where your derivative formula is valid and evaluate $C'(1/4)$.

(d) Let $y = \operatorname{arcsec}(x)$, with $|x| \geq 1$ and $0 \leq y \leq \pi$, $y \neq \pi/2$. In the following steps, work with $|x| > 1$.

(i) Starting from $\sec y = x$, use implicit differentiation to find $y' = \frac{1}{\sec y \tan y}$.

(ii) Use the Pythagorean identity to express $\tan y$ in terms of x . Explain why we need both branches of the square root, depending on the value of y .

(iii) Solve for y' in terms of x . Your formula should work for all $|x| > 1$.