

Worksheet 1: Limits and Continuity

Math 140: Calculus with Analytic Geometry

Name: _____

Date: _____

Directions. Show enough work to support each answer. Give exact values. You may use the limit laws, the fundamental trigonometric limits, and the Squeeze Theorem.

1. Evaluating limits.

Evaluate each limit. Name the main algebraic technique or theorem you use.

(a) $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4}$

(b) $\lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2}{x}$

(c) $\lim_{x \rightarrow 0} x^2 \cos\left(\frac{1}{x}\right)$

For part (c), begin with $-1 \leq \cos(1/x) \leq 1$ and write explicit lower and upper bounds before applying the Squeeze Theorem.

2. Continuity and types of discontinuities.

The table gives information about four functions at the indicated number c . For each row, decide whether the function is continuous at c . If it is not continuous, classify the discontinuity as *removable*, *jump*, or *infinite*, and identify which condition in the definition of continuity fails.

Case 1. $f_1(c) = 2$, $\lim_{x \rightarrow c^-} f_1(x) = 2$, and $\lim_{x \rightarrow c^+} f_1(x) = 2$.

Case 2. $f_2(c) = 4$, $\lim_{x \rightarrow c^-} f_2(x) = 3$, and $\lim_{x \rightarrow c^+} f_2(x) = 3$.

Case 3. $f_3(c) = 1$, $\lim_{x \rightarrow c^-} f_3(x) = -1$, and $\lim_{x \rightarrow c^+} f_3(x) = 1$.

Case 4. $f_4(c)$ is undefined, $\lim_{x \rightarrow c^-} f_4(x) = -\infty$, and $\lim_{x \rightarrow c^+} f_4(x) = +\infty$.

For any removable discontinuity, state how the value at c could be redefined to make the function continuous there.

3. Choosing a value to ensure continuity.

Let

$$g(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x < 2, \\ kx, & x \geq 2. \end{cases}$$

Find the value of k that makes g continuous at $x = 2$. Then explain why your choice makes g continuous for every real number.