

Worksheet 2: Derivative Rules

Math 140: Calculus with Analytic Geometry

Name: _____

Date: _____

Directions. Show enough work to support each answer. Give exact values. State the derivative rule or rules you use, and preserve any domain restrictions from the original function.

1. Deriving the derivative of the sine function.

Let $f(x) = \sin x$, where angles are measured in radians. Use the limit definition to derive a formula for $f'(x)$.

(a) Begin with

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}.$$

Use the identity

$$\sin(x+h) = \sin x \cos h + \cos x \sin h$$

to expand the numerator.

(b) Rearrange the difference quotient so that it has the form

$$\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right).$$

(c) Use the fundamental trigonometric limits

$$\lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} = 0$$

to find $f'(x)$.

2. Applying the derivative rules.

Find the derivative of each function. State the rule or rules you use, and simplify your final answer.

(a) $f(x) = 4x^5 - 3x^2 + 7x - 9$

(b) $g(x) = (x^2 + 1)(x^3 - 2x)$

(c) $p(x) = \frac{x^3 - 2x}{x^2 + 1}$

3. Using a derivative to study tangent lines.

Let

$$f(x) = \frac{x}{x^2 + 1}.$$

- (a) Use the quotient rule to find $f'(x)$.
- (b) Find an equation of the tangent line to the graph of f at $x = 0$.
- (c) Find all points on the graph of f where the tangent line is horizontal.

For part (c), explain how the zeros of f' identify the requested points.