

Worksheet 4: Related Rates

Math 140: Calculus with Analytic Geometry

Name: _____

Date: _____

Directions. Define your variables and label a sketch when no figure is supplied. Write an equation relating the changing quantities, differentiate with respect to time, and evaluate at the requested instant. Include units and explain the sign of each rate. Use radians for angles and give exact answers. Throughout this worksheet, a prime denotes differentiation with respect to time; for example, $h' = dh/dt$.

Topics. The topic titles indicate the types of related-rates problems you will be expected to solve. Each question explores one example of its topic; other problems may use different shapes or situations. Focus on understanding how the quantities are related and how their rates follow from those relationships.

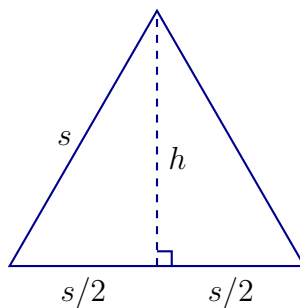
1. Changing Areas

A square panel expands while remaining square. At one instant its side length s is 8 cm and increases at 1 cm/s.

- (a) Sketch and label the square. Express its area A , perimeter P , and diagonal length d in terms of s . Use the Pythagorean theorem to obtain the diagonal formula.
- (b) Find the area rate and perimeter rate at this instant. Include units.
- (c) Find the diagonal rate. Explain why it differs from the side-length rate even though both lengths are measured in centimeters.

2. Changing Dimensions

An equilateral triangular panel expands while remaining equilateral. Its side length s increases at 2 cm/s. Let h denote its altitude.



- (a) Use one of the right triangles to express h in terms of s . Then express the panel's area in terms of s alone.
- (b) Find the height rate and area rate when $s = 10$ cm.
- (c) If the side length continues increasing at 2 cm/s, compare the height rate and area rate when $s = 20$ cm with those when $s = 10$ cm. Explain why one rate stays the same while the other changes.

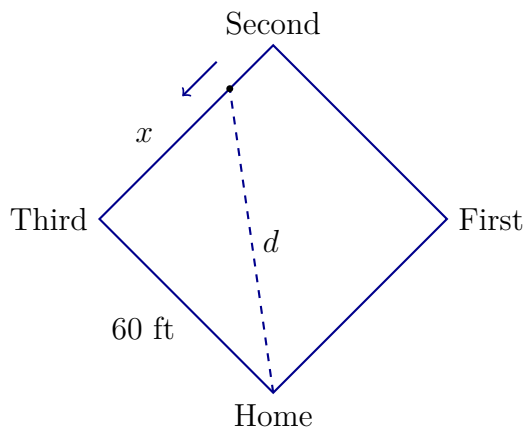
3. Changing Volumes

A rectangular tank has a fixed base measuring 4 m by 3 m and vertical sides. Water enters at $3 \text{ m}^3/\text{min}$ with no outflow. Assume that the water surface is horizontal and the tank is not yet full. Let h be the water depth and V the volume of water.

- (a) Sketch the water-filled portion and label its dimensions. Express V in terms of h . Which dimensions are constant, and which quantity changes?
- (b) Differentiate your equation with respect to time and find the water-level rate. Does this rate depend on the current depth?
- (c) At another time, water still enters at $3 \text{ m}^3/\text{min}$, but a drain releases water at $1 \text{ m}^3/\text{min}$. Find the net volume rate and the new water-level rate. Is the level rising or falling?

4. Changing Distances

A softball diamond is a square with side length 60 ft. A runner travels from second base toward third base at a constant speed of 20 ft/s. Let x be the runner's distance from third base and d the runner's straight-line distance from home plate.



- (a) Write a relationship between d and x . What is $\frac{dx}{dt}$, including its sign?
- (b) At what rate is the runner's distance from home plate changing when the runner is 45 ft from third base?
- (c) As the runner approaches third base, does the magnitude of the distance rate approach 20 ft/s, 0, or some other value? Explain using your formula.

5. Changing Angles

A camera at ground level is 3000 ft horizontally from a rocket's launchpad. The rocket rises vertically at 500 ft/s. Let y be its height, d its distance from the camera, and θ the camera's angle of elevation.

- (a) Draw and label the right triangle. Express $\sin \theta$, $\cos \theta$, and $\tan \theta$ as ratios of its side lengths.
- (b) How fast is d increasing when the rocket is 4000 ft high?
- (c) How fast must θ increase at this instant? Give the answer in radians per second.

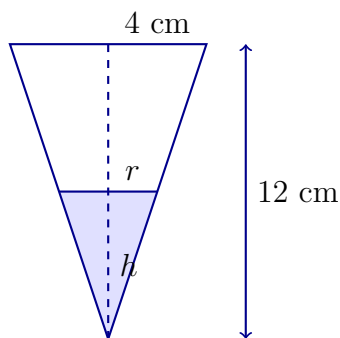
6. Changing Volume and Surface Area

A spherical ice ball melts while remaining spherical. Its volume decreases at a constant rate of $8\pi \text{ cm}^3/\text{min}$. Let r be its radius, V its volume, and S its surface area.

- (a) Sketch the ice ball and label its radius. Write formulas for V and S in terms of r . Differentiate the volume formula and find the radius rate when $r = 2 \text{ cm}$.
- (b) Find the surface-area rate at the same instant. Explain the signs of the radius and surface-area rates.
- (c) Is the radius decreasing faster when $r = 1 \text{ cm}$ or when $r = 2 \text{ cm}$? Support your comparison with an equation relating r' to r .

7. Changing Water Levels

A conical filter has its vertex pointing down, height 12 cm, and top radius 4 cm. Liquid drains at $3\pi \text{ cm}^3/\text{min}$ with no inflow. Let h be the liquid depth and r its surface radius. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.



- (a) Use similar triangles to express r in terms of h . Rewrite the liquid volume using only h .
- (b) Find h' when $8\pi \text{ cm}^3$ of liquid remains. First determine the liquid depth at this instant.
- (c) Find r' at the same instant. Why would setting $r = 4$ before differentiation give the wrong model?