Worksheet 7: Antiderivatives and Area

Math 140: Calculus with Analytic Geometry

Name: _____ **Date:** _____

Directions. Show your work, give exact answers, and label sketches clearly. For definite integrals, use geometry and integral properties. No external resources are allowed.

1. **Finding antiderivatives**

   (a) Evaluate the indefinite integral for $x > 0$:

   $$\int \left(6x^2 - 3\sqrt{x} + \frac{2}{x}\right)\, dx.$$

   Verify your answer by differentiation.

   (b) Evaluate the indefinite integral:

   $$\int \left(2e^x - 3\cos x + \frac{4}{1+x^2}\right)\, dx.$$

   Verify your answer by differentiation.

   (c) Find the function F on $(-1, 1)$ satisfying

   $$F'(x) = \cos x + \frac{1}{\sqrt{1-x^2}}, \qquad F(0) = 2.$$

   Check both conditions.

2. **Exact definite integrals from geometry**

(a) Sketch $y = 3 - x$ on $[1, 5]$ and evaluate

$$\int_1^5 (3 - x)\, dx.$$

Also find the total geometric area between the graph and the x-axis on this interval. Explain the difference between these two quantities.

(b) Identify and sketch the geometric region represented by

$$\int_1^5 \sqrt{4 - (x - 3)^2}\, dx.$$

State the circle's center and radius, and evaluate the integral exactly.

(c) Use geometry and properties of definite integrals to evaluate

$$\int_5^3 \left(2 + \sqrt{4 - (x - 3)^2}\right)\, dx.$$

Show how you handle the constant term, the portion of the circle, and the reversed limits.

3. **Left endpoint rectangles**

For Problems 3–5, approximate the area under $f(x) = 27 - x^3$ and above the x-axis on $[1, 3]$, using four equal-width subintervals.

(a) **Partition.** Divide $[1, 3]$ into four equal subintervals. Compute Δx, list the partition points $x_0, \ldots, x_4$, and write the four subintervals. List the left endpoints that will determine the rectangle heights.

(b) **Rectangles.** Evaluate f at each sample point from part (a). Sketch the curve and the four left rectangles on the same axes. Label the partition points and mark the point on the curve that determines each height.

(c) **Finite summation.** Write A_4^{left} in sigma notation with four terms, expand it as an explicit sum, and compute the approximation exactly. Show where the rectangle width enters your calculation.

4. **Right endpoint rectangles**

(a) **Partition.** Divide $[1, 3]$ into four equal subintervals. Compute Δx, list the partition points $x_0, \ldots, x_4$, and write the four subintervals. List the right endpoints that will determine the rectangle heights.

(b) **Rectangles.** Evaluate f at each sample point from part (a). Sketch the curve and the four right rectangles on the same axes. Label the partition points and mark the point on the curve that determines each height. The rectangle with height zero should be shown as a segment on the x-axis.

(c) **Finite summation.** Write A_4^{right} in sigma notation with four terms, expand it as an explicit sum, and compute the approximation exactly. Show where the rectangle width enters your calculation.

5. **Midpoint rectangles**

(a) **Partition.** Divide $[1, 3]$ into four equal subintervals. Compute Δx, list the partition points $x_0, \ldots, x_4$, and write the four subintervals. List the midpoints that will determine the rectangle heights.

(b) **Rectangles.** Evaluate f at each sample point from part (a). Sketch the curve and the four midpoint rectangles on the same axes. Label the partition points and mark the point on the curve that determines each height.

(c) **Finite summation.** Write A_4^{midpoint} in sigma notation with four terms, expand it as an explicit sum, and compute the approximation exactly. Show where the rectangle width enters your calculation.