Worksheet 8: Riemann Sums and the FTC

Math 140: Calculus with Analytic Geometry

Name _____ **Date** _____

Directions. Show your work and give exact answers. Use equal-width subintervals throughout.

1. **A limit of right endpoint sums**

   Evaluate $\int_1^2 (x^2 - x + 1)\,dx$ using the limit of A_n^{right}.

   (a) For n equal subintervals, find Δx and the right endpoint x_i of the ith subinterval. Write A_n^{right} in summation notation and expand the summand.

   (b) Use the formulas for $\sum_{i=1}^{n} 1$, $\sum_{i=1}^{n} i$, and $\sum_{i=1}^{n} i^2$ to write A_n^{right} in terms of n.

   (c) Find $\lim_{n\to\infty} A_n^{\text{right}}$. Explain why this limit equals the integral.

2. **The Riemann integral and a telescoping sum**

   Evaluate $\int_1^4 \frac{1}{x^2}\,dx$ by choosing sample points that make the Riemann sum telescope.

   (a) State the limit formula for $\int_a^b f(x)\,dx$ when f is continuous on $[a, b]$. Specify Δx, the partition points x_i, and the allowed choices of c_i. Does the limit depend on those choices?

   (b) Apply the Mean Value Theorem to $F(x) = -1/x$ on $[x_{i-1}, x_i]$ in the partition of $[1, 4]$. Set $F'(c_i)$ equal to the secant slope and solve for $c_i \in (x_{i-1}, x_i)$.

   (c) For this choice of c_i, write $\Delta x / c_i^2$ as a difference of reciprocals. Show how the sum telescopes, then take its limit to evaluate the integral.

3. **Fundamental Theorem of Calculus**

(a) State the evaluation formula of the Fundamental Theorem of Calculus, including its hypotheses on f and its antiderivative F.

(b) Partition $[a, b]$ into n equal subintervals. Explain why the Mean Value Theorem applies to F on $[x_{i-1}, x_i]$, where it places c_i, and how it gives $f(c_i)\,\Delta x = F(x_i) - F(x_{i-1})$.

(c) Sum the identities from part (b). Identify the terms that cancel and the terms that remain. Explain why taking the limit proves the evaluation formula.

4. **Evaluating definite integrals with the FTC**

Use the Fundamental Theorem of Calculus to evaluate each integral. Show the antiderivative and use vertical-bar notation to evaluate at the endpoints.

(a) $\displaystyle\int_1^4 \left(\sqrt{x} + \frac{1}{x}\right) dx$

(b) $\displaystyle\int_0^1 e^{2x}\, dx$

(c) $\displaystyle\int_{-\pi/2}^{\pi/6} \sin x\, dx$