

Math 140 Worksheet Week 12

Week 12: Definite Integrals

Instructions. Write clear solutions on your own paper. Show enough work to justify your answers. Upload a single PDF of your work to Canvas.

In this worksheet, we will study the quantity $\int_0^2 x^2 dx$. We will divide the interval $[0, 2]$ into n subintervals of equal width $\Delta x = \frac{2}{n}$. Over each subinterval, we will construct a rectangle. Then, we will use the area of these n rectangles to approximate the quantity $\int_0^2 x^2 dx$. Let A_n^{right} , A_n^{left} , and A_n^{midpoint} denote the approximate area using rectangles with height determined by the right, left, and midpoint, respectively, of each subinterval.

1. Consider the right endpoint rectangles.
 - a. Compute A_4^{right} .
 - b. Draw a picture that displays the curve $y = x^2$ and the four right endpoint rectangles used to approximate $\int_0^2 x^2 dx$.
 - c. Determine a formula for A_n^{right} that holds for any $n \geq 1$.
2. Consider the left endpoint rectangles.
 - a. Compute A_4^{left} .
 - b. Draw a picture that displays the curve $y = x^2$ and the four left endpoint rectangles used to approximate $\int_0^2 x^2 dx$.
 - c. Determine a formula for A_n^{left} that holds for any $n \geq 1$.
3. Consider the midpoint rectangles.
 - a. Compute A_4^{midpoint} .
 - b. Draw a picture that displays the curve $y = x^2$ and the four midpoint rectangles used to approximate $\int_0^2 x^2 dx$.
 - c. Determine a formula for A_n^{midpoint} that holds for any $n \geq 1$.