

# Math 140 Worksheet Week 13

## Week 13: Riemann Sums and Exact Area

**Instructions.** Write clear solutions on your own paper. Show enough work to justify your answers. Upload a single PDF of your work to Canvas.

In this worksheet, we will compute definite integrals using specially chosen sample points in Riemann sums. In each problem, the sample points  $c_i$  are chosen so that the Riemann sum telescopes.

For each problem:

- Divide the interval  $[a, b]$  into  $n$  subintervals of equal width  $\Delta x = \frac{b-a}{n}$ .
- Let  $x_i = a + i\Delta x$ .
- Use the given  $c_i$  values to form the Riemann sum

$$\sum_{i=1}^n f(c_i)\Delta x.$$

- Show that the sum telescopes and compute the exact value of the integral.

### 1. (Linear Function)

Evaluate

$$\int_a^b x \, dx.$$

Use the sample points

$$c_i = \frac{x_{i-1} + x_i}{2}.$$

### 2. (Exponential Function)

Evaluate

$$\int_a^b e^x \, dx.$$

Use the sample points

$$c_i = x_{i-1} + \ln\left(\frac{e^{\Delta x} - 1}{\Delta x}\right).$$

### 3. (Reciprocal Function)

Assume  $0 < a < b$ . Evaluate

$$\int_a^b \frac{1}{x} \, dx.$$

Use the sample points

$$c_i = \frac{x_i - x_{i-1}}{\ln x_i - \ln x_{i-1}}.$$

**4. (Reciprocal Square Function)**

Assume  $0 < a < b$ . Evaluate

$$\int_a^b \frac{1}{x^2} dx.$$

Use the sample points

$$c_i = \sqrt{x_{i-1}x_i}.$$

**5. (Reciprocal Square Root Function)**

Assume  $0 < a < b$ . Evaluate

$$\int_a^b \frac{1}{\sqrt{x}} dx.$$

Use the sample points

$$c_i = \left( \frac{\sqrt{x_{i-1}} + \sqrt{x_i}}{2} \right)^2.$$

**6. (Reflection)**

In each of the previous problems, the Riemann sum telescopes. Explain why this happens. What role does the function  $F(x)$  (an antiderivative of  $f(x)$ ) appear to play in this process?