Math 140 Worksheet 3 — Solution Key
Week 3 (through Wednesday)

1. To be continuous at $x = 2$, we need

$$\lim_{x \to 2^-} F(x) = \lim_{x \to 2^+} F(x) = F(2) = c.$$

Left limit: $\lim_{x \to 2^-}(x^2 - 1) = 2^2 - 1 = 3$.
Right limit: $\lim_{x \to 2^+}(3x - 5) = 3(2) - 5 = 1$.
Since $3 \neq 1$, the two-sided limit does *not* exist, so **no value of** c can make F continuous at $x = 2$.

2. (a) For $x \neq 3$,

$$r(x) = \frac{(x-3)(x+3)}{x-3} = x + 3.$$

(b) As written, r is not defined at $x = 3$, so it is not continuous at $x = 3$ (it has a removable discontinuity).

(c) Define $\tilde{r}(3) = 6$ (since $\lim_{x \to 3} r(x) = 3 + 3 = 6$) and $\tilde{r}(x) = r(x)$ for $x \neq 3$. Then $\tilde{r}$ is continuous for all real x.

3. (a) $p(2) = 8 - 14 + 1 = -5$ and $p(3) = 27 - 21 + 1 = 7$. Since $p(2) < 0 < p(3)$, there is a root in $(2, 3)$.

(b) $p(x)$ is a polynomial, so it is continuous on $[2, 3]$. Thus IVT applies.

4. (a) $f(0) = \cos 0 - 0 = 1$ and $f(1) = \cos 1 - 1 \approx 0.5403 - 1 < 0$. Since f is continuous on $[0, 1]$ and changes sign, IVT guarantees a zero in $(0, 1)$.

(b) Midpoint is $m = \frac{1}{2}$. Compute $f(\frac{1}{2}) = \cos(\frac{1}{2}) - \frac{1}{2} \approx 0.8776 - 0.5 > 0$. Since $f(\frac{1}{2}) > 0$ and $f(1) < 0$, the sign change occurs on $[\frac{1}{2}, 1]$, so the next interval is $[\frac{1}{2}, 1]$.

5. Let $g(x) = \sin x - \dfrac{x}{2}$, continuous on $[0, \pi]$. Compute $g(0) = 0 - 0 = 0$ and $g(\pi) = 0 - \dfrac{\pi}{2} < 0$. Also $g(\frac{\pi}{2}) = 1 - \frac{\pi}{4} > 0$. Since $g(\frac{\pi}{2}) > 0$ and $g(\pi) < 0$, by IVT there is a solution in $(\frac{\pi}{2}, \pi) \subset (0, \pi)$.