Math 140 Worksheet 6 — Solution Key

1.
$$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$
$$(x + 2y)\frac{dy}{dx} = -(2x + y) \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{2x + y}{x + 2y}$$

2. (a) $f'(x) = 2xe^{x^2}$

   (b) $g'(x) = \dfrac{6x}{3x^2 + 1}$

3.
$$y' = \frac{2}{\sqrt{1 - 4x^2}}$$

4. (a) $2x + 2y\frac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{x}{y}$

   (b) At $(1, 2)$, slope $= -\dfrac{1}{2}$

   Tangent line: $y - 2 = -\dfrac{1}{2}(x - 1)$

5. Applying implicit differentiation gives
$$\sec^2(y)\frac{dy}{dx} = 1.$$
Therefore,
$$\frac{dy}{dx} = \frac{1}{\sec^2(y)} = \frac{1}{x^2 + 1}.$$