Math 140 Worksheet 7

Week 7: Related Rates, Linear Approximations, and L'Hôpital's Rule

Instructions. Write clear solutions on your own paper. Show enough work to justify your answers. Upload a single PDF of your work to Canvas.

1. A liquid is draining at a rate of 3 cm^3/min from a cone filter with radius 4 cm and height 12 cm. Find the rate at which the height of the liquid in the cone is changing when there is 10 cm^3 liquid left. Note that the volume of liquid is given by $V = \frac{\pi}{3}r^2h$.

2. Use linear approximation to estimate

$$\sqrt{16.2}.$$

   Clearly identify the function, the point of linearization, and the linearization formula.

3. The volume of a cube is given by $V = s^3$, where s is the side length.

   (a) Use differentials to estimate the change in volume if the side length increases from 5 cm to 5.03 cm.

   (b) Estimate the percentage error in the volume.

4. Evaluate the limit. Clearly verify that L'Hôpital's Rule applies.

$$\lim_{x\to 0} \frac{\ln(1+4x)}{x}$$

5. Evaluate the limit.

$$\lim_{x\to 0^+} (1+3x)^{1/x}$$

   (Hint: Let $y = (1+3x)^{1/x}$, take the natural logarithm of both sides, and use L'Hôpital's Rule.)