Math 140 Worksheet 7 — Solution Key

1. For a sphere, $V = \frac{4}{3}\pi r^3$, so
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}.$$
At $r = 10$ and $\frac{dr}{dt} = 0.5$,
$$\frac{dV}{dt} = 4\pi(10)^2(0.5) = 200\pi \text{ cm}^3/\text{min}.$$

2. Let $f(x) = \sqrt{x}$ and choose $a = 16$. Then $f(16) = 4$ and
$$f'(x) = \frac{1}{2\sqrt{x}} \quad \Rightarrow \quad f'(16) = \frac{1}{8}.$$
Thus the linearization is
$$L(x) = 4 + \frac{1}{8}(x - 16).$$
So
$$\sqrt{16.2} \approx L(16.2) = 4 + \frac{1}{8}(0.2) = 4.025.$$

3. (a) $V = s^3$ so $dV = 3s^2\, ds$. With $s = 5$ and $ds = 0.03$,
$$dV = 3(5)^2(0.03) = 2.25 \text{ cm}^3.$$
(b) The volume at $s = 5$ is $V = 125$, so the estimated percent error is
$$\frac{|dV|}{V} \cdot 100\% = \frac{2.25}{125} \cdot 100\% = 1.8\%.$$

4. Substitution gives
$$\lim_{x\to 0} \frac{\ln(1+4x)}{x} = \frac{0}{0},$$
so L'Hôpital's Rule applies. Differentiate numerator and denominator:
$$\lim_{x\to 0} \frac{\ln(1+4x)}{x} = \lim_{x\to 0} \frac{\frac{4}{1+4x}}{1} = \lim_{x\to 0} \frac{4}{1+4x} = 4.$$

5. Let $y = (1+3x)^{1/x}$. Then
$$\ln y = \frac{\ln(1+3x)}{x}.$$
As $x \to 0^+$ this is $\frac{0}{0}$, so apply L'Hôpital's Rule:
$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\frac{3}{1+3x}}{1} = \lim_{x\to 0^+} \frac{3}{1+3x} = 3.$$
Therefore,
$$\lim_{x\to 0^+} y = e^3.$$