Math 140 Worksheet 8
Week 8: Rolle's Theorem, The Mean Value Theorem, and the First Derivative Test

Instructions. Write clear solutions on your own paper. Show enough work to justify your answers. Upload a single PDF of your work to Canvas.

1. Let $f(x) = x^3 - 6x^2 + 9x$ on the interval $[0, 3]$.

   (a) Verify that f satisfies the hypotheses of Rolle's Theorem on $[0, 3]$.

   (b) Find all values c in $(0, 3)$ guaranteed by Rolle's Theorem.

2. Let $f(x) = \ln(x)$ on the interval $[1, e^2]$.

   (a) Verify that f satisfies the hypotheses of the Mean Value Theorem on $[1, e^2]$.

   (b) Find all values c in $(1, e^2)$ such that

   $$f'(c) = \frac{f(e^2) - f(1)}{e^2 - 1}.$$

   (c) Give a geometric interpretation of your result.

3. Let $f(x) = x^3 - 3x^2 - 9x + 5$.

   (a) Find all critical numbers of f.

   (b) Determine the intervals on which f is increasing and decreasing.

   (c) Use the First Derivative Test to classify each critical point.

4. Let $f(x) = xe^{-x}$.

   (a) Find $f'(x)$.

   (b) Find all critical numbers.

   (c) Determine where f is increasing and decreasing.

   (d) Use the First Derivative Test to classify each critical point.

5. Let

   $$f(x) = x^3 e^{-x^2}.$$

   (a) Find all critical numbers of f.

   (b) Use the First Derivative Test to classify each critical point.

   (c) Compute $f''(x)$. What do you notice about the values of $f''(x)$ at each critical point?