HW 0 Solutions

Ch. 3 Rvw

118

$$y = \frac{(2x+1)^3(x^2-1)^2}{x+3}$$

$$\ln(y) = \ln\left(\frac{(2x+1)^3(x^2-1)^2}{x+3}\right)$$

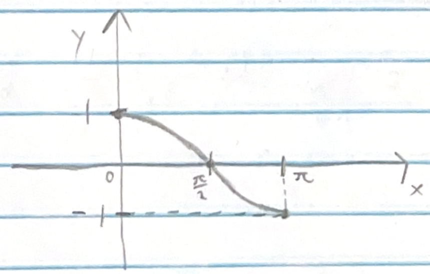
$$= 3\ln(2x+1) + 2\ln(x^2-1) - \ln(x+3)$$

$$\frac{1}{y}(y') = 3\left(\frac{1}{2x+1}\right)(2) + 2\left(\frac{1}{x^2-1}\right)(2x) - \left(\frac{1}{x+3}\right)$$

$$y' = y\left[\left(\frac{6}{2x+1}\right) + \left(\frac{4x}{x^2-1}\right) - \left(\frac{1}{x+3}\right)\right]$$

$$\boxed{y' = \left(\frac{(2x+1)^3(x^2-1)^2}{x+3}\right)\left(\frac{6}{2x+1} + \frac{4x}{x^2-1} - \frac{1}{x+3}\right)}$$

122 $f(x) = \cos(x)$, $0 \le x \le \pi$, $f\left(\frac{\pi}{2}\right) = 0$



Note that the graph of f passes the horizontal line test. Thus, f is one-to-one, so f^{-1} exists.

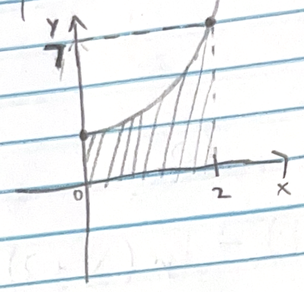
By Thm. 3.17, $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$

Note that $f'(x) = -\sin(x)$ and $f^{-1}(0) = \frac{\pi}{2}$.

$$\boxed{\text{Thus, } (f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))} = \frac{1}{-\sin\left(\frac{\pi}{2}\right)} = \frac{1}{-1} = -1}$$

Ch 5 Rvw.

28 $y = x^2 + 3 \quad [0, 2]$



$$\int_0^2 x^2 + 3 = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} (c_i^2 + 3)\Delta x_i$$

$$\Delta x_i = \frac{2-0}{n} = \frac{2}{n}$$

$$c_i = 0 + i\left(\frac{2}{n}\right) = \frac{2i}{n}$$

$$\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} (c_i^2 + 3)\Delta x_i = \lim_{n \to \infty} \sum_{i=1}^{n} \left(\left(\frac{2i}{n}\right)^2 + 3\right)\left(\frac{2}{n}\right)$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{4i^2}{n^2} + 3\right)\left(\frac{2}{n}\right) = \lim_{n \to \infty} \frac{2}{n}\left(\sum_{i=1}^{n} \frac{4i^2}{n^2} + \sum_{i=1}^{n} 3\right)$$

$$= \lim_{n \to \infty} \frac{2}{n}\left(\frac{4}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right) + 3n\right)$$

$$= \lim_{n \to \infty} \frac{8(2n^3 + 3n^2 + n)}{6n^3} + 6 = \lim_{n \to \infty} \frac{4(2n^2 + 3n + 1)}{3n^2} + 6$$

$$= \frac{4(2)}{3} + 6 = \frac{8}{3} + 6 = \boxed{\frac{26}{3}}$$

68 $\int_0^1 x^2(x^3-2)^3\,dx$

$u = x^3 - 2$

$\frac{du}{dx} = 3x^2$

$dx = \frac{1}{3x^2}\,du$

$\int x^2(x^3-2)^3\,dx = \int \frac{1}{3}u^3\,du = \frac{1}{3}\int u^3\,du = \frac{1}{3}\left(\frac{1}{4}u^4\right) + C$

$= \frac{1}{12}u^4 + C = \frac{1}{12}(x^3-2)^4 + C$

$\int_0^1 x^2(x^3-2)^3\,dx = \frac{1}{12}(x^3-2)^4 \Big|_0^1 = \frac{1}{12}\left[(1^3-2)^4 - (0^3-2)^4\right]$

$= \frac{1}{12}\left[(-1)^4 - (-2)^4\right] = \frac{1}{12}(1-16) = -\frac{15}{12} = \boxed{-\frac{5}{4}}$

72 $2\pi\int_{-1}^0 x^2\sqrt{x+1}\,dx$

$u = x+1$

$\frac{du}{dx} = 1$

$du = dx$

$\int x^2\sqrt{x+1}\,dx = \int (u-1)^2 u^{\frac{1}{2}}\,du = \int (u^2-2u+1)u^{\frac{1}{2}}\,du$

$= \int u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}}\,du = \int u^{\frac{5}{2}}\,du - 2\int u^{\frac{3}{2}}\,du + \int u^{\frac{1}{2}}\,du$

$= \frac{2}{7}u^{\frac{7}{2}} - 2\left(\frac{2}{5}u^{\frac{5}{2}}\right) + \frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{7}(x+1)^{\frac{7}{2}} - \frac{4}{5}(x+1)^{\frac{5}{2}}$

$+ \frac{2}{3}(x+1)^{\frac{3}{2}} + C$

$2\pi\int_{-1}^0 x^2\sqrt{x+1}\,dx = 2\pi\left(\frac{2}{7}(x+1)^{\frac{7}{2}} - \frac{4}{5}(x+1)^{\frac{5}{2}} + \frac{2}{3}(x+1)^{\frac{3}{2}}\right)\Big|_{-1}^0$

$= 2\pi\left[\frac{2}{7}(0+1)^{\frac{7}{2}} - \frac{4}{5}(0+1)^{\frac{5}{2}} + \frac{2}{3}(0+1)^{\frac{3}{2}} - \frac{2}{7}(-1+1)^{\frac{7}{2}} + \frac{4}{5}(-1+1)^{\frac{5}{2}}\right.$

$\left.- \frac{2}{3}(-1+1)^{\frac{3}{2}}\right] = 2\pi\left(\frac{2}{7} - \frac{4}{5} + \frac{2}{3} - 0\right) = \boxed{\frac{32\pi}{105}}$