MATH-141 **Name:** ____________________

Fall 2025

Exam III Worksheet

November 12 **Pledge:** ____________________

Each question topic and point value is recorded in the tables below. Note that this exam must be completed within the 50 minutes allotted. Also, you must work without any external resources, which includes no notes, calculator, nor any equivalent software. You must show an appropriate amount of work to justify your answer for each problem. If you run out of room for a given problem, you may continue your work on the back of the page. By writing your name and signing the pledge you are stating that you understand the rules outlining this exam.

Scoring Table

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Topics Table

Question	Topic
1	Taylor Polynomial
2	Taylor Series
3	Interval of Convergence
4	Operations on Power Series
5	Integration and Differentiation of Power Series

1. (a) (2 points) Let $f(x)$ be infinitely differentiable at x_0. State the definition of the nth Taylor polynomial of $f(x)$ at x_0. When $n = 1$, what line does the Taylor polynomial represent?

   (b) (3 points) Let $f(x) = \sqrt{x}$. Find the $n = 2$ Taylor polynomial of $f(x)$ at $x_0 = 1$.

   (c) (2 points) Let $f(x)$ be infinitely differentiable at x_0. State the integral remainder term for the nth Taylor polynomial. What is the bound on this remainder?

   (d) (3 points) Determine the bound on the Taylor polynomial from part (b) at $x = 2$.

2. (a) (2 points) Let $f(x)$ be infinitely differentiable at x_0. State the definition of the Taylor series of $f(x)$ at x_0. When is the Taylor series equal to $f(x)$?

(b) (4 points) Derive the Taylor series of $f(x) = e^{2x}$ at $x_0 = 0$.

(c) (4 points) Show that the Taylor series from (b) is equal to $f(x)$ for all x.

3. Determine the interval of convergence for each of the following power series.

(a) (5 points) $\displaystyle\sum_{k=0}^{\infty} \frac{3^k}{k!} x^k$

(b) (5 points) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k 2^k} (x-1)^k$

4. Consider the following power series representation

$$\frac{1}{1+x} = \sum_{k=0}^{\infty} (-1)^k x^k, \qquad -1 < x < 1.$$

Determine a power series representation for each of the following functions. State the interval of convergence for each series.

(a) (5 points) $\dfrac{2}{3x+4}$, centered at $x_0 = 1$.

(b) (5 points) $\dfrac{1}{x^2+3x+2}$, centered at $x_0 = 0$.

5. Consider the following power series representation

$$\ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} x^k, \qquad -1 < x \leq 1.$$

(a) (5 points) Use the power series to show that $\frac{d}{dx}\ln(1+x) = \frac{1}{1+x}$. Hint: Make use of the power series representation in problem 4.

(b) (5 points) Determine a power series representation of $\int \ln(1+x)dx$. State the interval of convergence.