MATH-141
Fall 2025
Final Exam Worksheet
December 12, 2025

Name: ______________________________

Pledge: ______________________________

Each question topic and point value is recorded in the tables below. Note that this exam must be completed within the 1 hour and 50 minutes allotted. Also, you must work without any external resources, with the exception of a double sided 3x5 note card. You must show an appropriate amount of work to justify your answer for each problem. If you run out of room for a given problem, you may continue your work on the back of the page. By writing your name and signing the pledge you are stating that you understand the rules outlining this exam.

Scoring Table

Question	Points	Score
1	8	
2	8	
3	12	
4	10	
5	6	
6	10	
7	10	
8	8	
9	10	
10	8	
Total:	90	

Topics Table

Question	Topic
1	Area between two curves
2	Volume of a solid of revolution
3	Integration methods
4	Comparison tests
5	Ratio and root tests
6	Taylor series
7	Interval of convergence
8	Operations on power series
9	Area and surface area in polar coordinates
10	Multivariable functions

1. (a) (2 points) Sketch the region bounded by the curves $y = 6 - x^2$ and $y = x$. Include points of intersection.

(b) (6 points) Find the area of the region.

2. (a) (2 points) Sketch the region bounded by the curves $y = \sqrt{x}$, $x = 1$, $x = 9$, and $y = 0$. Include points of intersection.

(b) (6 points) Find the volume of the solid formed by revolving the region about the x-axis.

3. Evaluate the following indefinite integrals.

(a) (4 points) $\int \sqrt{4 - x^2}\ dx$

(b) (4 points) $\int xe^x\ dx$

(c) (4 points) $\int \frac{1}{x(x^2 + 1)}\ dx$

4. (a) (4 points) Use the integral test to determine if $\sum_{k=1}^{\infty} \frac{1}{k^2}$ converges or diverges.

(b) (3 points) Use the direct comparison test to dermine if $\sum_{k=1}^{\infty} \frac{2}{k}$ converges or diverges.

(c) (3 points) Use the limit comparison test to determine if $\sum_{k=1}^{\infty} \frac{k+1}{5k^3+4k+3}$ converges or diverges.

5. Use the ratio or root test to determine if the following series converge or diverge.

(a) (3 points) $\displaystyle\sum_{k=1}^{\infty} \frac{k!}{3^k}$

(b) (3 points) $\displaystyle\sum_{k=1}^{\infty} \frac{e^k}{k^k}$

6. (a) (2 points) Let $f(x)$ be infinitely differentiable at x_0. State the definition of the Taylor series of $f(x)$ at x_0. When is the Taylor series equal to $f(x)$?

(b) (4 points) Derive the Taylor series of $f(x) = \cos(x)$ at $x_0 = 0$. Write your answer using sigma notation.

(c) (4 points) Show that the Taylor series from (b) is equal to $f(x)$ for all x.

7. Determine the interval of convergence for each of the following power series.

(a) (3 points) $\displaystyle\sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}$

(b) (4 points) $\displaystyle\sum_{k=0}^{\infty} (-1)^k \left(\frac{x-1}{2}\right)^k$

(c) (3 points) $\displaystyle\sum_{k=1}^{\infty} k^k x^k$

8. Consider the following power series representation

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

(a) (2 points) Determine a power series representation (centered at 0) for $\frac{1}{1+x}$

(b) (6 points) Using part (a) and the integral, determine a power series representation (centred at 0) of $\ln(1+x)$.

9. Consider the curve described by the polar equation

$$r = 1 + \cos(\theta),\ 0 \leq \theta \leq \pi.$$

(a) (2 points) Sketch the graph of the curve.

(b) (4 points) Find the area of the interior of the curve.

(c) (4 points) Find the surface area of the solid formed by revolving the curve about the x-axis.

10. Consider the multivariable function

$$f(x, y) = y^2 - x^2$$

(a) (2 points) Sketch the contour plot of $f(x, y)$ using level curves of height $c = -1, 0, 1$.

(b) (2 points) Use the traces $x = 0$, $y = 0$, and $z = 0$ to sketch the graph of $z = f(x, y)$.

(c) (4 points) Find the equation of the line tangent to $z = f(x, y)$ at the point $(x, y) = (1, 2)$ that is parallel to the x-axis.