Calculus of Parametric Equations Worksheet

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Exercises

I. Consider the parabola with vertex $(3, 2)$ and directrix $x = 0$.

    (a) Find an equation for this parabola in the Cartesian coordinate system.

    (b) Find a parameterization of this parabola.

    (c) Use your parameterization to find the tangent line at the point $(6, 8)$.

II. Consider the ellipse with center $(1, 1)$, foci $(1, 1)$, and semimajor axis length 2.

    (a) Find an equation for this ellipse in the Cartesian coordinate system.

    (b) Find a parameterization of this ellipse.

    (c) Use your parameterization to find the arc length of this ellipse.

III. Consider the hyperbola with center $(0, 0)$, foci $(0, 5)$, and vertex $(0, 4)$.

    (a) Find an equation for this hyperbola in the Cartesian coordinate system.

    (b) Find a parameterization for the upper half of the hyperbola.

    (c) Suppose the upper half of the hyperbola from $(-3, \sqrt{32})$ to $(3, \sqrt{(32)})$ is revolved around the x-axis to form a surface of revolution. Use your parameterization to find the surface area of this surface of revolution.