Power Series Worksheet

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1 Exercises

I. Find a power series representation of $\int e^{x^2} dx$. What is the radius of convergence for this series?

II. Find a power series representation of $\ln\left(\frac{1+x}{1-x}\right)$. What is the interval of convergence for this series? Explain why this series can be used to approximate $\ln(y)$ for all $y > 0$.

III. Find a power series representation of $\arctan(x)$. What is the interval of convergence for this series? Explain why this series can be used to approximate π.

IV. Find a power series representation for the hyperbolic sine function, $\sinh(x) = \frac{e^x - e^{-x}}{2}$. What is the radius of convergence for this series?

V. Find a power series representation for the hyperbolic cosine function, $\cosh(x) = \frac{e^x + e^{-x}}{2}$ What is the radius of convergence for this series?

VI. Use the power series representations to show that $\frac{d}{dx}\sinh(x) = \cosh(x)$ and $\frac{d}{dx}\cosh(x) = \sinh(x)$.