Calculus with Analytic Geometry II

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1 Power Series Worksheet

Find the radius and interval of convergence for each of the following power series.

I. $\displaystyle\sum_{k=0}^{\infty}(-1)^k\frac{(x+1)^k}{2^k}$

II. $\displaystyle\sum_{k=0}^{\infty}\frac{x^k}{k!}$

III. $\displaystyle\sum_{k=0}^{\infty}\left(\frac{x-1}{3}\right)^k$

Using the power series representation

$$\frac{1}{1-x}=\sum_{k=0}^{\infty},$$

with radius of convergence 1 and interval of convergence $(-1,1)$, determine a power series representation of the following functions. For each power series representation, determine the radius and interval of convergence.

I. $\displaystyle\frac{1}{1+x}$

II. $\displaystyle\frac{1}{1+x^2}$

III. $\arctan(x)$

Next, determine a power series representation with indicated center for each of the following functions.

I. $\displaystyle\frac{3}{2x-1}, \qquad x_0=2$

II. $\displaystyle\frac{4x}{x^2+2x-3}, \qquad x_0=0$

III. $\ln(1-x^2), \qquad x_0=0$