

Taylor Series Worksheet

Thomas R. Cameron

November 7, 2025

1 Exercises

Using the definition, find the n th Taylor polynomial for each of the following functions. Write your answer using sigma notation.

- I. $f(x) = e^x$, centered at $x_0 = 0$.
- II. $f(x) = \ln(x)$, centered at $x_0 = 1$.
- III. $f(x) = \cos(x)$, centered at $x_0 = 0$.
- IV. $f(x) = \sinh(x)$, centered at $x_0 = 0$.

For each of the n th Taylor polynomials above, identify what values of x have a remainder that approaches zero as n approaches infinity.

2 Bonus

Consider the following function

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

- (a) Find $f'(x)$ and $f''(x)$.
- (b) Explain why $f^{(k)}(0) = 0$ for all $k \geq 0$.
- (c) Find the Taylor series of $f(x)$ centered at 0.
- (d) What is the interval of convergence for this series? What does this series converge to? What does that say about the Taylor polynomial remainder?