Differential Equations

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1 Daily Quiz

Find the formula for u_1' and u_2' in variation of parameters applied to

$$y'' + 3y' + 2y = \frac{1}{1 + e^t}.$$

2 Key Topics

Today we continue to use the method of variation of parameters to find a particular solution of the second-order linear non-homogeneous differential equation:

$$y'' + p(t)y' + q(t)y = f(t), \tag{1}$$

where $y := y(t)$ is a differentiable function. For further reading, see [2, Section 5.7] or [1, Section 4.2].

2.1 Variation of Parameters

Let $\{y_1, y_2\}$ form a fundamental set for the complementary homogeneous equation of (1). We seek a particular of the form

$$y_p = u_1 y_1 + u_2 y_2, \tag{2}$$

where $u_1 := u_1(t)$ and $u_2 := u_2(t)$ are differentiable functions. Recall that we derived the following formulas for the derivative of u_1 and u_2:

$$u_1' = -\frac{y_2 f(t)}{W(y_1, y_2)}$$

$$u_2' = \frac{y_1 f(t)}{W(y_1, y_2)}$$

3 Exercises

Find the general solution for each of the following differential equations. Note that in some cases it may be wise to use the method of undetermined coefficients.

   I. $y'' + 3y' + 2y = \frac{1}{1+e^t}$

  II. $y'' - 5y' + 6y = 3te^t$

 III. $y'' + y = \csc(t)$

 IV. $y'' - 4y' = 4t + 3$

References

[1] T. W. Judson, *The Ordinary Differential Equations Project*, Creative Commons Attribution-Noncommercial-Share Alike, 1st ed., 2023.

[2] W. Trench, *Elementary Differential Equations with Boundary Value Problems*, Creative Commons Attribution-Noncommercial-Share Alike, 1st ed., 2013.