Differential Equations

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1 Daily Quiz

2 Key Topics

Today, we review the Laplace transform and the inverse Laplace transform. For further reading, see [1, Sections 8.1-8.2].

Suppose that $f(t)$ is piecewise continuous and exponentially bounded, i.e., $|f(t)| \leq Me^{at}$ for $t \geq t_0$, where M, a, t_0 are positive constants. Then, the Laplace transform of $f(t)$ is defined as

$$\mathcal{L}(f(t)) = F(s) = \int_0^\infty e^{-st} f(t) dt$$

The inverse Laplace transform is defined as

$$\mathcal{L}^{-1}(F(s)) = f(t).$$

Below is a brief table of values for the Laplace transform and its inverse. Note that $u_c(t)$ denotes the unit step function for $c \geq 0$.

$f(t)$	$F(s) = \mathcal{L}(f(t))$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$\cos(\omega t)$	$\frac{s}{\omega^2+s^2}$
$\sin(\omega t)$	$\frac{\omega}{\omega^2+s^2}$
$\cosh(\omega t)$	$\frac{s}{s^2-w^2}$
$\sinh(\omega t)$	$\frac{w}{s^2-w^2}$
$e^{at}f(t)$	$F(s-a)$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$

Table 1: Laplace Transforms

3 Exercises

1. Use the definition to find the Laplace transform of $\sinh(\omega t)$. **Hint:** $\sinh(\omega t) = \frac{e^{\omega t} - e^{-\omega t}}{2}$.

2. Find the inverse Laplace transform of

$$F(s) = \frac{s^2 + s + 1}{(s-1)(s+2)\left(s^2 + 4s + 5\right)}$$

Hint: Use partial fraction decomposition to write $F(s) = \frac{A}{s-1} + \frac{B}{s+2} + \frac{Cs+D}{s^2+4s+5}$. Then, complete the square on $s^2 + 4s + 5$.

References

[1] W. Trench, *Elementary Differential Equations with Boundary Value Problems*, Creative Commons Attribution-Noncommercial-Share Alike, 1st ed., 2013.